Sinais contínuos

$$G_T(t) = u(t+T/2) - u(t-T/2), \quad \delta(t) = \frac{d}{dt}u(t) \ , \quad u(t) = \int_{-\infty}^{t} \delta(\beta)d\beta \ , \quad \int_{-\infty}^{+\infty} f(t)\delta(t)dt = f(0)$$

Laplace (bilateral):

$$H(s) = \mathcal{L}\{h(t)\} = \int_{-\infty}^{+\infty} h(t)\exp(-st)dt \quad , \quad s \in \Omega_h \quad , \quad \int_{-\infty}^{+\infty} x(t)dt = X(s)\Big|_{s=0, \ \ 0\in\Omega_x}$$

$$\mathcal{L}\{\delta(t)\} = 1, \ s \in \mathbb{C} \quad , \quad \mathcal{L}\{x(t) = x_1(t) * x_2(t)\} = \mathcal{L}\{x_1(t)\}\mathcal{L}\{x_2(t)\} \quad , \quad \Omega_x = \Omega_{x_1} \cap \Omega_{x_2}$$

$$\mathcal{L}\{y(t) = x(t-\tau)\} = X(s)\exp(-s\tau) \quad , \quad \Omega_y = \Omega_x \quad , \quad \mathcal{L}\{\exp(-at)u(t)\} = \frac{1}{s+a} \quad , \quad \text{Re}(s+a) > 0$$

$$\mathcal{L}\{\exp(-\alpha t)\cos(\beta t)u(t)\} = \frac{(s+\alpha)}{(s+\alpha)^2+\beta^2} \ , \mathcal{L}\{\exp(-\alpha t)\text{sen}(\beta t)u(t)\} = \frac{\beta}{(s+\alpha)^2+\beta^2} \ , \ \text{Re}(s+\alpha) > 0$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}\exp(-at)u(t)\Big\} = \frac{1}{(s+a)^{m+1}} \ , \ \ \text{Re}(s+a) > 0 \ , \ m \in \mathbb{N}$$

$$\mathcal{L}\Big\{y(t) = \int_{-\infty}^{t} x(\beta)u(\beta)d\beta\Big\} = \frac{1}{s}\mathcal{L}\{x(t)\} \quad , \quad \Omega_y \supset \Omega_x \cap \{s \in \mathbb{C} : \text{Re}(s) > 0\}$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}u(t)\Big\} = \frac{1}{s^{m+1}} \ , \ \ \text{Re}(s) > 0 \ , \ m \in \mathbb{N} \quad , \quad \mathcal{L}\{x(-t)\} = X(-s) \quad , \quad -s \in \Omega_x$$

$$\mathcal{L}\{y(t) = \exp(-at)x(t)\} = X(s+a) \quad ; \quad \Omega_y = (s+a) \in \Omega_x$$

$$\mathcal{L}\{y(t) = t^m x(t)\} = (-1)^m \frac{d^m X(s)}{ds^m} \quad , \quad \Omega_y = \Omega_x \ , \ m \in \mathbb{N} \quad , \quad \mathcal{L}\{\dot{x}(t)\} = sX(s) \quad , \quad \Omega_{\dot{x}} \supset \Omega_x$$

Laplace (unilateral)

$$\mathcal{L}\{x(t)\} = \int_{0}^{+\infty} x(t)\exp(-st)dt, \quad \mathcal{L}\{\dot{x}(t)\} = s\mathcal{L}\{x(t)\} - x(0) \quad , \quad s \in \Omega_x$$

$$\mathcal{L}\left\{x^{(m)}(t) = \frac{d^m x(t)}{dt^m}\right\} = s^m \mathcal{L}\{x(t)\} - \sum_{k=0}^{m-1} s^{m-k-1}x^{(k)}(0)$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}\exp(-at)u(t)\Big\} = \frac{1}{(s+a)^{m+1}} \ , \ \ \text{Re}(s+a) > 0 \ , \ m \in \mathbb{N}$$

$$\mathcal{L}\{\cos(\beta t)\exp(-at)u(t)\} = \frac{s+a}{(s+a)^2+\beta^2} \quad , \ \text{Re}(s+a) > 0$$

$$\mathcal{L}\{\text{sen}(\beta t)\exp(-at)u(t)\} = \frac{\beta}{(s+a)^2+\beta^2} \quad , \ \text{Re}(s+a) > 0$$

$$x(0^+) = \lim_{t\to 0^+} x(t) = \lim_{s\to+\infty} sX(s) \ , \quad \lim_{t\to+\infty} x(t) = \lim_{s\to 0} sX(s)$$

Coeficientes a determinar (equações diferenciais)

$$D(p)y(t) = 0 \quad \Rightarrow \quad y(t) = \sum_{k=1}^{m} a_k f_k(t), \quad f_k(t) \text{ modos próprios (considerando multiplicidades)}$$

Se λ é raiz de multiplicidade r de $D(\lambda)$, então $\exp(\lambda t)$, $t\exp(\lambda t)$, $\ldots$, $t^{r-1}\exp(\lambda t)$ são modos próprios.

$$D(p)y(t) = N(p)x(t) \;\;,\;\; \text{se } \bar{D}(p)x(t) = 0 \text{ então } \bar{D}(p)D(p)y(t) = 0$$

$$\text{Solução forçada:} \quad y(t) = y_h(t) + y_f(t) \quad \Rightarrow \quad D(p)y_f(t) = N(p)x(t) \;,\; D(p)y_h(t) = 0$$

$$y_f(t) = \sum_{k=1}^{m} b_k g_k(t), \quad g_k(t) \text{ modos forçados (considerando multiplicidades e ressonâncias)}$$

Var. estado: $\dot{v}(t) = f(v(t), x(t), t)$, $y(t) = g(v(t), x(t), t)$ Pontos de equilíbrio: $\bar{v} : f(\bar{v}, \bar{x}) = 0$, $\bar{x}$ =cte.

Sistema linear (em torno dos pontos de equilíbrio) $A = \left[\dfrac{\partial f_i}{\partial v_j}\right]\Big|_{\bar{v},\bar{x}}$, $B = \left[\dfrac{\partial f_i}{\partial x_j}\right]\Big|_{\bar{v},\bar{x}}$, $C = \left[\dfrac{\partial g_i}{\partial v_j}\right]\Big|_{\bar{v},\bar{x}}$, $D = \left[\dfrac{\partial g_i}{\partial x_j}\right]\Big|_{\bar{v},\bar{x}}$

$$\frac{N(p)}{D(p)} = \frac{\beta_2 p^2 + \beta_1 p + \beta_0}{p^3 + \alpha_2 p^2 + \alpha_1 p + \alpha_0} + \beta_3, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; c = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 \end{bmatrix}, d = \begin{bmatrix} \beta_3 \end{bmatrix}$$

$$\dot{v} = Av + bx \;,\; y = cv + dx \;,\; \frac{N(p)}{D(p)} = c(p\mathrm{I} - A)^{-1}b + d = b'(p\mathrm{I} - A')^{-1}c' + d$$

$$v = T\hat{v} \quad \Rightarrow \quad \hat{A} = T^{-1}AT, \; \hat{b} = T^{-1}b, \hat{c} = cT \quad , \quad T \text{ não singular}$$

A representação entrada-saída é invariante com transformações de similaridade.

Solução da equação de estado

$$y(t) = c\exp(At)v_0 + c\big(\exp(At)u(t)\big) * (bx(t)) + dx(t), \; Y(s) = c(s\mathrm{I} - A)^{-1}v_0 + \big(c(s\mathrm{I} - A)^{-1}b + d\big)X(s)$$

$$\text{Cayley-Hamilton:} \quad \Delta(\lambda) = \det(s\mathrm{I} - A) = 0 \quad \Rightarrow \quad \Delta(A) = 0$$

$$f(\lambda) = \sum_{i=0}^{n-1} \rho_i \lambda^i \;,\; \Delta(\lambda) = 0 \quad \Rightarrow \quad f(A) = \sum_{i=0}^{n-1} \rho_i A^i \;,\; f\big(\mathrm{diag}(A_1, \ldots, A_N)\big) = \mathrm{diag}\big(f(A_1), \ldots, f(A_N)\big)$$

$$\text{Bloco de Jordan: } J_k(\sigma) = \begin{bmatrix} \sigma & 1 & \cdots & 0 \\ 0 & \sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma \end{bmatrix} \;,\; f(J_k(\sigma)) = \begin{bmatrix} f(\lambda) & \dot{f}(\lambda) & \cdots & f^{(k-1)}(\lambda)/(k-1)! \\ 0 & f(\lambda) & \cdots & f^{(k-2)}(\lambda)/(k-2)! \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f(\lambda) \end{bmatrix}_{\lambda=\sigma}$$

$$\text{Forma modal: } M = \begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix}, \; \Delta(\lambda) = (\lambda - \sigma)^2 + \omega^2, \quad \exp(Mt) = \exp(\sigma t) \begin{bmatrix} \cos(\omega t) & -\mathrm{sen}(\omega t) \\ \mathrm{sen}(\omega t) & \cos(\omega t) \end{bmatrix}$$

$$\text{Forma modal de Jordan: } \begin{bmatrix} M & \mathrm{I} & \cdots & 0 \\ 0 & M & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M \end{bmatrix}, \quad \exp\left(\begin{bmatrix} M & \mathrm{I} \\ 0 & M \end{bmatrix} t \right) = t\exp(\sigma t) \begin{bmatrix} \cos(\omega t) & -\mathrm{sen}(\omega t) \\ \mathrm{sen}(\omega t) & \cos(\omega t) \end{bmatrix}$$

Forma de Jordan de $A \in \mathbb{R}^{n \times n}$, $\nu(M_\lambda) = n - \mathrm{rank}(M_\lambda)$ (dimensão do espaço nulo de $M_\lambda = A - \lambda\mathrm{I}$ = número de blocos de Jordan associados a λ = multiplicidade geométrica do autovalor λ): $AQ = QJ$, $J = Q^{-1}AQ$, Q formada por autovetores linearmente independentes e autovetores generalizados.

$$\dot{v} = Av + bx \;,\; y = cv + dx, v(0) \quad , \quad \text{para } x \text{ solução de } x = \bar{c}\bar{v} \;,\; \dot{\bar{v}} = \bar{A}\bar{v}, \bar{v}(0)$$

$$\Rightarrow \quad \text{Sistema autônomo aumentado: } \begin{bmatrix} \dot{v} \\ \dot{\bar{v}} \end{bmatrix} = \begin{bmatrix} A & b\bar{c} \\ 0 & \bar{A} \end{bmatrix} \begin{bmatrix} v \\ \bar{v} \end{bmatrix} \quad , \quad \begin{bmatrix} v(0) \\ \bar{v}(0) \end{bmatrix} = \begin{bmatrix} v_0 \\ \bar{v}_0 \end{bmatrix} \quad , \quad y = \begin{bmatrix} c & d\bar{c} \end{bmatrix} \begin{bmatrix} v \\ \bar{v} \end{bmatrix}$$