Sinais discretos

degrau: $u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$, impulso: $\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$, $\delta[n] = u[n] - u[n-1]$, $u[n] = \sum_{k=-\infty}^{n} \delta[k]$

Convolução: $x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$, $x[n] * \delta[n] = x[n]$, $x[n] * \delta[n-m] = x[n-m]$

SLIT

$\Rightarrow$ $y[n] = x[n]*h[n]$, $h[n] = \mathcal{G}\{\delta[n]\}$, $y[n] = z^n*h[n] = H(z)z^n$, $H(z) = \sum_{k=-\infty}^{+\infty} h[k]z^{-k} = \mathcal{Z}\{h[n]\}$

Resp. em freqüência:

$M(\omega)\exp(j\phi(\omega)) = H(z = \exp(j\omega))$, $h[n]$ real , $x[n] = \cos(\omega n)$ $\Rightarrow$ $y[n] = M(\omega)\cos(\omega n + \phi(\omega))$

Transf. Z: $\mathcal{Z}\{x[n]\} = \sum_{k=-\infty}^{+\infty} x[k]z^{-k}$, $\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}, |z| > |a|$, $\mathcal{Z}\{-a^n u[-n-1]\} = \frac{z}{z-a}, |z| < |a|$

$$\mathcal{Z}\{na^{n-1}u[n]\} = \frac{z}{(z-a)^2} \ , \ |z| > |a| \quad , \quad \mathcal{Z}\{-na^{n-1}u[-n]\} = \frac{z}{(z-a)^2} \ , \ |z| < |a|$$

$$\mathcal{Z}\{x[n]\} = X(z), z \in \Omega_x \ \Leftrightarrow \ \mathcal{Z}\{x[-n]\} = X(z^{-1}), z^{-1} \in \Omega_x \quad , \quad \mathcal{Z}\{x_1[n] * x_2[n]\} = \mathcal{Z}\{x_1[n]\}\mathcal{Z}\{x_2[n]\}$$

$$m \in \mathbb{Z}_+ : \quad \mathcal{Z}\{n^m x[n]\} = \left(-z\frac{d}{dz}\right)^m X(z) \quad , \quad \sum_{k=-\infty}^{+\infty} k^m x[k] = \mathcal{Z}\{n^m x[n]\}\Big|_{z=1} \ , \ 1 \in \Omega_x$$

$$\mathcal{Z}\{y[n] = x[n-m]u[n-m]\} = z^{-m}\mathcal{Z}\{x[n]u[n]\}, \quad \mathcal{Z}\{x[n+m]u[n]\} = z^m\Big(\mathcal{Z}\{x[n]u[n]\} - \sum_{k=0}^{m-1} x[k]z^{-k}\Big)$$

$$\mathcal{Z}\left\{\begin{pmatrix} n \\ m \end{pmatrix} a^{n-m}u[n]\right\} = \frac{z}{(z-a)^{m+1}} \ , \ |z| > |a| \ , \ m \in \mathbb{N} \quad , \quad \mathcal{Z}\{n^2 a^n u[n]\} = \frac{az^2 + a^2 z}{(z-a)^3} \ , \ |z| > |a|$$

$$\mathcal{Z}\left\{\begin{pmatrix} n+m \\ m \end{pmatrix} a^n u[n]\right\} = (1 - az^{-1})^{-(m+1)} = \frac{z^{m+1}}{(z-a)^{m+1}} \ , \ m \in \mathbb{N} \ , \ |z| > |a|$$

$$x[0] = \lim_{|z| \to +\infty} X(z) \ , \ \Omega_x \text{ exterior de um círculo} \quad , \quad x[+\infty] = \lim_{z \to 1}(z-1)X(z) \ , \ |z| > \rho \ , \ 0 < \rho \leq 1$$

Transf. Z e Probabilidade: $G_{\mathbb{X}}(z) = \mathcal{E}\{z^{\mathbb{X}}\} = \mathcal{Z}\{p[n]\} = \sum_{k=-\infty}^{+\infty} p[k]z^k = \sum_{k=-\infty}^{+\infty} \Pr\{\mathbb{X} = k\}z^k$

$$G_{\mathbb{X}}(z) = \sum_{n=0}^{+\infty} \frac{1}{n!}\frac{d^n}{dz^n}G_{\mathbb{X}}(z)\Big|_{z=0} z^n, \quad \mathbb{X}, \mathbb{Y} \text{ var. aleat. independentes} \ \Rightarrow \ \mathcal{E}\{z^{(\mathbb{X}+a\mathbb{Y})}\} = \mathcal{E}\{z^{\mathbb{X}}\}\mathcal{E}\{(z^a)^{\mathbb{Y}}\}$$

$$\mathcal{E}\{\mathbb{X}\} = \sum_k kp[k] \ , \quad \mathcal{E}\{\mathbb{X}^m\} = \sum_k k^m p[k] \ , \quad \sigma_{\mathbb{X}}^2 = \mathcal{E}\{\mathbb{X}^2\} - \mathcal{E}\{\mathbb{X}\}^2 \ , \quad \mathcal{E}\{\mathbb{X}^m\} = \left(\frac{zd}{dz}\right)^m \mathcal{Z}\{p[n]\}\Big|_{z=1}$$

Eq. dif. (Transf. Z): $\mathcal{Z}\{y[n+2]u[n]\} = z^2Y(z) - z^2y[0] - zy[1]$, $\mathcal{Z}\{y[n+1]u[n]\} = zY(z) - zy[0]$

Eq. dif. (Coef. a determinar): $py[n] \triangleq y[n+1]$, **Autofunção** (SLIT): $x[n] = z^n \Rightarrow y_f[n] = H(z)z^n$
$D(p)y[n] = 0$ $\Rightarrow$ $y[n] = \sum_{k=1}^{m} a_k f_k[n]$, $f_k[n]$ modos próprios (considerando multiplicidades)

λ: raiz de multiplicidade r de $D(\lambda) \Rightarrow \lambda^n$, $n\lambda^n$, $\ldots$, $n^{r-1}\lambda^n$ (r modos próprios)

$$D(p)y[n] = N(p)x[n] \ , \ \text{se } \bar{D}(p)x[n] = 0 \text{ então } \bar{D}(p)D(p)y[n] = 0$$

Solução forçada: $y[n] = y_h[n] + y_f[n]$ $\Rightarrow$ $D(p)y_f[n] = N(p)x[n]$, $D(p)y_h[n] = 0$

$y_f[n] = \sum_{k=1}^{m} b_k g_k[n]$, $g_k[n]$ modos forçados (considerando multiplicidades e ressonâncias)