Sinais discretos

Convolução: $x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$, $x[n] * \delta[n] = x[n]$, $x[n] * \delta[n-m] = x[n-m]$

SLIT

$\Rightarrow$ $y[n] = x[n]*h[n]$, $h[n] = \mathcal{G}\{\delta[n]\}$, $y[n] = z^n * h[n] = H(z)z^n$, $H(z) = \sum_{k=-\infty}^{+\infty} h[k]z^{-k} = \mathcal{Z}\{h[n]\}$

Resp. em frequência:

$M(\omega)\exp(j\phi(\omega)) = H(z = \exp(j\omega))$, $h[n]$ real , $x[n] = \cos(\omega n)$ $\Rightarrow$ $y[n] = M(\omega)\cos(\omega n + \phi(\omega))$

Transformada Z

$\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$, $|z| > |a|$, $\mathcal{Z}\{-a^n u[-n-1]\} = \frac{z}{z-a}$, $|z| < |a|$

$\mathcal{Z}\{na^{n-1}u[n]\} = \frac{z}{(z-a)^2}$, $|z| > |a|$, $\mathcal{Z}\{-na^{n-1}u[-n]\} = \frac{z}{(z-a)^2}$, $|z| < |a|$

$\mathcal{Z}\{x[n]\} = X(z), z \in \Omega_x \Leftrightarrow \mathcal{Z}\{x[-n]\} = X(z^{-1}), z^{-1} \in \Omega_x$, $\mathcal{Z}\{x_1[n] * x_2[n]\} = \mathcal{Z}\{x_1[n]\}\mathcal{Z}\{x_2[n]\}$

$\mathcal{Z}\{n^m x[n]\} = \left(-z\frac{d}{dz}\right)^m X(z)$, $\sum_{k=-\infty}^{+\infty} k^m x[k] = \mathcal{Z}\{n^m x[n]\}\Big|_{z=1}$, $1 \in \Omega_x$, $m \in \mathbb{N}$

$\mathcal{Z}\{y[n] = x[n-m]u[n-m]\} = z^{-m}\mathcal{Z}\{x[n]u[n]\}$, $m \in \mathbb{Z}_+$, $\Omega_y = \Omega_x$

$\mathcal{Z}\{x[n+m]u[n]\} = z^m\Big(\mathcal{Z}\{x[n]u[n]\} - \sum_{k=0}^{m-1} x[k]z^{-k}\Big)$, $m \in \mathbb{Z}_+$

$\mathcal{Z}\left\{\begin{pmatrix} n \\ m \end{pmatrix} a^{n-m}u[n]\right\} = \frac{z}{(z-a)^{m+1}}$, $|z| > |a|$, $m \in \mathbb{N}$, $\mathcal{Z}\{na^n u[n]\} = \frac{az}{(z-a)^2}$, $|z| > |a|$

$\mathcal{Z}\left\{\begin{pmatrix} n+m \\ m \end{pmatrix} a^n u[n]\right\} = (1 - az^{-1})^{-(m+1)} = \frac{z^{m+1}}{(z-a)^{m+1}}$, $m \in \mathbb{N}$, $|z| > |a|$

$x[0] = \lim_{|z|\to+\infty} X(z)$, Ω_x exterior de um círculo , $x[+\infty] = \lim_{z\to 1}(z-1)X(z)$, $|z| > \rho$, $0 < \rho \le 1$

Transformada Z aplicada a probabilidade

$G_{\mathbb{X}}(z) = \mathcal{E}\{z^{\mathbb{X}}\} = \mathcal{Z}\{p[n]\} = \sum_{k=-\infty}^{+\infty} p[k]z^k = \sum_{k=-\infty}^{+\infty} \Pr\{\mathbb{X} = k\}z^k$

Sequências $p[n]$ à direita do 0: $G_{\mathbb{X}}(z) = \sum_{n=0}^{+\infty} \frac{1}{n!}\frac{d^n}{dz^n}G_{\mathbb{X}}(z)\Big|_{z=0} z^n$

$\mathcal{E}\{\mathbb{X}\} = \sum_k kp[k]$, $\sigma^2_{\mathbb{X}} = \mathcal{E}\{\mathbb{X}^2\} - \mathcal{E}\{\mathbb{X}\}^2$, $\mathcal{E}\{\mathbb{X}^m\} = \left(\frac{zd}{dz}\right)^m \mathcal{Z}\{p[n]\}\Big|_{z=1}$

$\mathbb{X}$, $\mathbb{Y}$ var. aleatórias independentes $\Rightarrow$ $\mathcal{E}\{z^{(\mathbb{X}+\mathbb{Y})}\} = \mathcal{E}\{z^{\mathbb{X}}\}\mathcal{E}\{z^{\mathbb{Y}}\}$

$\mathbb{X}$, $\mathbb{Y}$ var. aleatórias independentes $\Rightarrow$ $\mathcal{E}\{z^{(a\mathbb{X}+b\mathbb{Y})}\} = \mathcal{E}\{(z^a)^{\mathbb{X}}\}\mathcal{E}\{(z^b)^{\mathbb{Y}}\}$

Coeficientes a determinar (equações a diferenças)

$$D(p)y[n] = 0 \quad \Rightarrow \quad y[n] = \sum_{k=1}^{m} a_k f_k[n] \quad f_k[n] \text{ modos próprios (considerando multiplicidades)}$$

Se λ é raiz de multiplicidade r de $D(\lambda)$, então λ^n, $n\lambda^n$, $\ldots$, $n^{r-1}\lambda^n$ são modos próprios.

$$D(p)y[n] = N(p)x[n] \ \ , \ \text{ se } \ \bar{D}(p)x[n] = 0 \text{ então } \ \bar{D}(p)D(p)y[n] = 0$$

$$\text{Solução forçada:} \quad y[n] = y_h[n] + y_f[n] \quad \Rightarrow \quad D(p)y_f[n] = N(p)x[n] \ , \ D(p)y_h[n] = 0$$

$y_f[n]$: combinação linear dos modos forçados (considerando multiplicidades e ressonâncias)

Sinais contínuos

$$G_T(t) = u(t+T/2) - u(t-T/2), \quad \delta(t) = \frac{d}{dt}u(t) \ , \quad u(t) = \int_{-\infty}^{t} \delta(\beta)d\beta \ , \quad \int_{-\infty}^{+\infty} f(t)\delta(t)dt = f(0)$$

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\beta)x_2(t-\beta)d\beta \ , \quad x(t) * \delta(t) = x(t) \ , \quad x(t) * u(t) = \mathcal{I}_x(t) = \int_{-\infty}^{t} x(\beta)d\beta$$

$$\mathcal{I}_{x*y}(t) = x(t) * \mathcal{I}_y(t) = \mathcal{I}_x(t) * y(t) = u(t) * x(t) * y(t) \ , \quad \frac{d}{dt}\big(x(t) * y(t)\big) = \dot{x}(t) * y(t) = x(t) * \dot{y}(t)$$

SLIT: $y(t) = x(t) * h(t), \ h(t) = \mathcal{G}\{\delta(t)\}$

$$y(t) = \exp(st) * h(t) = H(s)\exp(st), \ H(s) = \int_{-\infty}^{+\infty} h(t)\exp(-st)dt = \mathcal{L}\{h(t)\}, \ s \in \Omega_h$$

$$\mathcal{L}\{\exp(-at)u(t)\} = \frac{1}{s+a}, \ \text{Re}(s+a) > 0, \quad \mathcal{L}\{y(t) = x(t-\tau)\} = X(s)\exp(-s\tau), \ \Omega_y = \Omega_x$$

Sinais ortogonais: $\big\langle x(t)y^*(t)\big\rangle = \int_{-\infty}^{+\infty} x(t)y^*(t)dt = 0$, Projeção ortogonal: $\big\langle \epsilon(t)g_k^*(t)\big\rangle = 0$, $\forall k$

Gram-Schmidt $g_1(t) = f_1(t)$; $\quad g_k(t) = f_k(t) - \sum_{\ell=1}^{k-1} \frac{\big\langle f_k(t)g_\ell(t)\big\rangle}{\big\langle g_\ell^2(t)\big\rangle} g_\ell(t) \quad , \ k = 2, \ldots, n$

Série de Fourier

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k \exp(jk\omega_0 t) \ \Leftrightarrow \ c_k = \frac{1}{T}\int_T x(t)\exp(-jk\omega_0 t)dt \ , \quad \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |c_k|^2 \text{ (potência média)}$$

$$\mathcal{F}_S\{x(t)\}_T = \{c_k\}_{\omega_0} \quad \Rightarrow \quad c_0 = \frac{1}{T}\int_T x(t)dt \ \text{ (valor médio)} \ , \ x(0) = \sum_{k=-\infty}^{+\infty} c_k$$

$$\mathcal{F}_S\Big\{\frac{d}{dt}x(t)\Big\}_T = \{jk\omega_0 c_k\}_{\omega_0} \ , \quad \mathcal{F}_S\Big\{\int_{-\infty}^{t} x(\beta)d\beta\Big\}_T = \Big\{\frac{1}{jk\omega_0}c_k\Big\}_{\omega_0} (x(t) \text{ com valor médio } 0)$$

$$x(t) \text{ real:} \qquad x(t) = a_0 + \sum_{k=1}^{+\infty}\big(a_k\cos(k\omega_0 t) + b_k \text{sen}(k\omega_0 t)\big)$$

$$a_0 = c_0 = \frac{1}{T}\int_T x(t)dt \ , \ a_k = (c_k + c_{-k}) = \frac{2}{T}\int_T x(t)\cos(k\omega_0 t)dt \ , \ b_k = j(c_k - c_{-k}) = \frac{2}{T}\int_T x(t)\text{sen}(k\omega_0 t)dt$$