degrau: $u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$, impulso: $\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$, $\delta[n] = u[n] - u[n-1]$, $u[n] = \sum_{k=-\infty}^{n} \delta[k]$

Transf. Z: $\mathcal{Z}\{x[n]\} = \sum_{k=-\infty}^{+\infty} x[k]z^{-k}$, $\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}, |z| > |a|$, $\mathcal{Z}\{-a^n u[-n-1]\} = \frac{z}{z-a}, |z| < |a|$

$\mathcal{Z}\{na^{n-1}u[n]\} = \frac{z}{(z-a)^2}$, $|z| > |a|$, $\mathcal{Z}\{-na^{n-1}u[-n]\} = \frac{z}{(z-a)^2}$, $|z| < |a|$

$\mathcal{Z}\{x[n]\} = X(z), z \in \Omega_x \Leftrightarrow \mathcal{Z}\{x[-n]\} = X(z^{-1}), z^{-1} \in \Omega_x$, $\mathcal{Z}\{x_1[n] * x_2[n]\} = \mathcal{Z}\{x_1[n]\}\mathcal{Z}\{x_2[n]\}$

$m \in \mathbb{Z}_+ :$ $\mathcal{Z}\{n^m x[n]\} = \left(-z\frac{d}{dz}\right)^m X(z)$, $\sum_{k=-\infty}^{+\infty} k^m x[k] = \mathcal{Z}\{n^m x[n]\}\Big|_{z=1}$, $1 \in \Omega_x$

$\mathcal{Z}\{y[n] = x[n-m]u[n-m]\} = z^{-m}\mathcal{Z}\{x[n]u[n]\}$, $\mathcal{Z}\{x[n+m]u[n]\} = z^m\Big(\mathcal{Z}\{x[n]u[n]\} - \sum_{k=0}^{m-1} x[k]z^{-k}\Big)$

$\mathcal{Z}\left\{\begin{pmatrix} n \\ m \end{pmatrix} a^{n-m}u[n]\right\} = \frac{z}{(z-a)^{m+1}}$, $|z| > |a|$, $m \in \mathbb{N}$, $\mathcal{Z}\{n^2 a^n u[n]\} = \frac{az^2 + a^2 z}{(z-a)^3}$, $|z| > |a|$

$\mathcal{Z}\left\{\begin{pmatrix} n+m \\ m \end{pmatrix} a^n u[n]\right\} = (1 - az^{-1})^{-(m+1)} = \frac{z^{m+1}}{(z-a)^{m+1}}$, $m \in \mathbb{N}$, $|z| > |a|$

$x[0] = \lim_{|z| \to +\infty} X(z)$, Ω_x exterior de um círculo , $x[+\infty] = \lim_{z \to 1}(z-1)X(z)$, $|z| > \rho$, $0 < \rho \leq 1$

Transf. Z e Probabilidade: $G_{\mathbb{X}}(z) = \mathcal{E}\{z^{\mathbb{X}}\} = \mathcal{Z}\{p[n]\} = \sum_{k=-\infty}^{+\infty} p[k]z^k = \sum_{k=-\infty}^{+\infty} \Pr\{\mathbb{X} = k\}z^k$

$G_{\mathbb{X}}(z) = \sum_{n=0}^{+\infty} \frac{1}{n!}\frac{d^n}{dz^n}G_{\mathbb{X}}(z)\Big|_{z=0} z^n$, $\mathbb{X}, \mathbb{Y}$ var. aleat. independentes $\Rightarrow$ $\mathcal{E}\{z^{(\mathbb{X}+a\mathbb{Y})}\} = \mathcal{E}\{z^{\mathbb{X}}\}\mathcal{E}\{(z^a)^{\mathbb{Y}}\}$

$\mathcal{E}\{\mathbb{X}\} = \sum_k kp[k]$, $\mathcal{E}\{\mathbb{X}^m\} = \sum_k k^m p[k]$, $\sigma_{\mathbb{X}}^2 = \mathcal{E}\{\mathbb{X}^2\} - \mathcal{E}\{\mathbb{X}\}^2$, $\mathcal{E}\{\mathbb{X}^m\} = \left(\frac{zd}{dz}\right)^m \mathcal{Z}\{p[n]\}\Big|_{z=1}$

Eq. dif. (Transf. Z): $\mathcal{Z}\{y[n+2]u[n]\} = z^2 Y(z) - z^2 y[0] - zy[1]$, $\mathcal{Z}\{y[n+1]u[n]\} = zY(z) - zy[0]$

Eq. dif. (Coef. a determinar): $py[n] \triangleq y[n+1]$, **Autofunção** (SLIT): $x[n] = z^n \Rightarrow y_f[n] = H(z)z^n$

$D(p)y[n] = 0$ $\Rightarrow$ $y[n] = \sum_{k=1}^{m} a_k f_k[n]$, $f_k[n]$ modos próprios (considerando multiplicidades)

λ: raiz de multiplicidade r de $D(\lambda) \Rightarrow \lambda^n, n\lambda^n, \ldots, n^{r-1}\lambda^n$ (r modos próprios)

$$D(p)y[n] = N(p)x[n] \ , \text{ se } \bar{D}(p)x[n] = 0 \text{ então } \bar{D}(p)D(p)y[n] = 0$$

Solução forçada: $y[n] = y_h[n] + y_f[n]$ $\Rightarrow$ $D(p)y_f[n] = N(p)x[n]$, $D(p)y_h[n] = 0$

$y_f[n] = \sum_{k=1}^{m} b_k g_k[n]$, $g_k[n]$ modos forçados (considerando multiplicidades e ressonâncias)

Variáveis de estado: $\dot{v}(t) = f(v(t), x(t), t)$, $y(t) = g(v(t), x(t), t)$

Ponto de equilíbrio: $\bar{v}$ tal que $f(\bar{v}, \bar{x}) = 0$, $\bar{x}$ =cte. Sistema linear (em torno de $\bar{v}$)

$$A = \left[\frac{\partial f_i}{\partial v_j}\right]\Bigg|_{\bar{v},\bar{x}}, \ B = \left[\frac{\partial f_i}{\partial x_j}\right]\Bigg|_{\bar{v},\bar{x}}, \ C = \left[\frac{\partial g_i}{\partial v_j}\right]\Bigg|_{\bar{v},\bar{x}}, \ D = \left[\frac{\partial g_i}{\partial x_j}\right]\Bigg|_{\bar{v},\bar{x}} \quad \begin{cases} \text{Autovalores: } \det(\lambda I - A) = 0 \\ \exists i \mid \mathrm{Re}(\lambda_i) > 0 : \bar{v} \text{ instável} \\ \mathrm{Re}(\lambda_i) < 0, \forall i : \bar{v} \text{ assint. estável} \\ \exists i \mid \mathrm{Re}(\lambda_i) = 0 : \bar{v} \text{ inconclusivo} \end{cases}$$

$$p = \frac{d}{dt}, \quad \frac{N(p)}{D(p)} = \frac{\overline{\beta}_2 p^2 + \overline{\beta}_1 p + \overline{\beta}_0}{p^3 + \alpha_2 p^2 + \alpha_1 p + \alpha_0} + \beta_3 = c(p\mathrm{I} - A)^{-1}b + d = b'(p\mathrm{I} - A')^{-1}c' + d$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad A' = \begin{bmatrix} 0 & 0 & -\alpha_0 \\ 1 & 0 & -\alpha_1 \\ 0 & 1 & -\alpha_2 \end{bmatrix}, \quad c' = \begin{bmatrix} \overline{\beta}_0 \\ \overline{\beta}_1 \\ \overline{\beta}_2 \end{bmatrix},$$

$$c = \begin{bmatrix} \overline{\beta}_0 & \overline{\beta}_1 & \overline{\beta}_2 \end{bmatrix}, \qquad d = \begin{bmatrix} \beta_3 \end{bmatrix} \qquad b' = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \qquad d = \begin{bmatrix} \beta_3 \end{bmatrix}$$

$$\dot{v} = Av + bx \ , \ y = cv + dx \ , \ \frac{N(p)}{D(p)} = c(p\mathrm{I} - A)^{-1}b + d = b'(p\mathrm{I} - A')^{-1}c' + d$$

$$v = T\hat{v} \quad \Rightarrow \quad \hat{A} = T^{-1}AT, \ \hat{b} = T^{-1}b, \ \hat{c} = cT \ , \ T \text{ não singular}$$