Resolução PR2 - 2S - 2017:

Questão 1:

$$\dot{v} = v(v-1)(v+2) = v^3 + v^2 - 2v$$

a) Pontos de equilíbrio?

$$\dot{v} = 0 = v(v-1)(v+2) \rightarrow \begin{cases} v = 0 \\ v = 1 \\ v = -2 \end{cases}$$

Pontos de equilíbrio

(0), (1), (-2)

b) Aproximação linear e comportamento local

$$\dot{v} = \frac{d\dot{v}}{dv} \cdot v = \left[3v^2 + 2v - 2\right] \cdot v$$

Em $v = 0$	Em $v = 1$	Em $v = -2$
$\dot{v} = -2v$	$\dot{v} = 3v$	$\dot{v} = 6v$
autovalor = -2	autovalor = 3	autovalor = 6
As. estável	Instável	instável

Questão 2 :

$$\dot{v}_1 = v_2(v_2-1)(v_1-2) - 3u$$
$$\dot{v}_2 = v_1(v_1+2)(v_2+1) + 2u^2$$

a) Pontos de equilíbrio para $u = 0$

$$\dot{v}_1 = 0 = v_2(v_2-1)(v_1-2) \to v_2 = 0,\quad v_2 = 1,\quad v_1 = 2$$
$$\dot{v}_2 = 0 = v_1(v_1+2)(v_2+1) \to v_1 = 0,\quad v_1 = -2,\quad v_2 = -1$$

• Se $v_2 = 0 \to v_1 = 0$ $(0,0) \leftarrow$
$\to v_1 = -2$ $(-2,0) \leftarrow$

• Se $v_2 = 1 \to v_1 = 0$ $(0,1) \leftarrow$
$\to v_1 = -2$ $(-2,1) \leftarrow$

• Se $v_1 = 2 \to v_2 = -1$ $(2,-1) \leftarrow$

• Se $v_2 = -1 \to v_1 = 2$ $(2,-1)$ igual

• Se $v_1 = 0 \to$ $v_2 = 0$ $(0,0)$ igual
$v_2 = 1$ $(0,1)$ igual

• Se $v_1 = -2 \to$ $v_2 = 0$ $(-2,0)$ igual
$v_2 = 1$ $(-2,1)$ igual

Pontos de eq.

$(0,0),\ (-2,0),\ (0,1),\ (-2,1),\ (2,-1)$

b) sistema linearizado

$$\dot{v} = Av + bx$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial \dot{v}_1}{\partial v_1} & \frac{\partial \dot{v}_1}{\partial v_2} \\ \frac{\partial \dot{v}_2}{\partial v_1} & \frac{\partial \dot{v}_2}{\partial v_2} \end{bmatrix}}_{A} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\partial \dot{v}_1}{\partial x} \\ \frac{\partial \dot{v}_2}{\partial x} \end{bmatrix}}_{b} x$$

$$\frac{\partial \dot{v}_1}{\partial v_1} = v_2^2 - v_2 \qquad \frac{\partial \dot{v}_1}{\partial x} = -3$$

$$\frac{\partial \dot{v}_1}{\partial v_2} = 2v_1v_2 - v_1 - 4v_2 + 2 \qquad \frac{\partial \dot{v}_2}{\partial x} = 4x$$

$$\frac{\partial \dot{v}_2}{\partial v_1} = 2 + 2v_1v_2 + 2v_1 + 2v_2$$

$$\frac{\partial \dot{v}_2}{\partial v_2} = v_1^2 + 2v_1$$

$$\boxed{A = \begin{bmatrix} v_2^2 - v_2 & 2v_1v_2 - v_1 - 4v_2 + 2 \\ 2v_1v_2 + 2 + 2v_1 + 2v_2 & v_1^2 + 2v_1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ 4x \end{bmatrix}}$$

Questão 3: Realização (A, b, c, d)?

$$(p^3 - 7p^2 - 6p - 5)\, y(t) = (3p^3 - 22p^2 - 20p - 18)\, x(t)$$

$$\frac{y(t)}{x(t)} = \frac{3p^3 - 22p^2 - 20p - 18}{p^3 - 7p^2 - 6p - 5}$$ CASO PRÓPRIO

$$\frac{y(t)}{x(t)} = \underbrace{3}_{d} + \frac{-\underbrace{p^2}_{\bar\beta_2} \underbrace{-2p}_{\bar\beta_1} - \underbrace{3}_{\bar\beta_0}}{p^3 \underbrace{-7p^2}_{\alpha_2} \underbrace{-6p}_{\alpha_1} \underbrace{-5}_{\alpha_0}}$$

$$\begin{cases} \bar\beta_0 = -3 \\ \bar\beta_1 = -2 \\ \bar\beta_2 = -1 \end{cases} \qquad \begin{cases} \alpha_0 = -5 \\ \alpha_1 = -6 \\ \alpha_2 = -7 \end{cases}$$

FORMA

$$\dot v = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x$$

$$y = [\beta_0 \;\; \beta_1 \;\; \beta_2]\, v + [d]\, x$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & 6 & 7 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [-3 \;\; -2 \;\; -1] \qquad d = [3]$$

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Questão 4:

$$\dot{v} = \underbrace{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}}_{A} v \qquad v(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad y = \underbrace{[1\ \ 2]}_{c}\, v$$

a) $Y(s) = ?$

Def: $Y(s) = c(sI - A)^{-1}\left(b\cancel{X(s)}^{0} + v(0)\right) + D\cancel{X(s)}^{0}$

$$Y(s) = c(sI - A)^{-1} v(0)$$

$$= [1\ \ 2] \underbrace{\begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\frac{1}{s^2 + 5s + 6} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$Y(s) = \frac{1}{(s+2)(s+3)} \cdot [1\ \ 2] \begin{bmatrix} 2s + 11 \\ s - 12 \end{bmatrix}$$

$$Y(s) = \frac{4s - 13}{(s+2)(s+3)} = \frac{A^{=-21}}{(s+2)} + \frac{B^{=25}}{(s+3)}$$

Transformada

$\mathcal{L}\{e^{at}\mu(t)\} = \frac{1}{s-a}$

$$\boxed{Y(s) = \frac{-21}{(s+2)} + \frac{25}{(s+3)}}$$

b) $\boxed{y(t) = \left(-21e^{-2t} + 25e^{-3t}\right)\mu(t)}$

Questão 5: $A = ?$

$$A^{-4} + 2A^{-3} + 3A^{-2} + 4A^{-1} = I$$

$$\Delta(A) = 0 \longleftarrow \Delta(\lambda) = 0$$

$$(A^4)\ A^{-4} + 2A^{-3} + 3A^{-2} + 4A^{-1} - I = 0\ (A^4)$$

$$A^4 - 4A^3 - 3A^2 - 2A - I = 0$$

$$\downarrow$$

$$\lambda^4 \underbrace{-4}_{\alpha_3}\lambda^3 \underbrace{-3}_{\alpha_2}\lambda^2 \underbrace{-2}_{\alpha_1}\lambda \underbrace{-1}_{\alpha_0} = \Delta(\lambda) \quad \begin{cases} \alpha_0 = -1 \\ \alpha_1 = -2 \\ \alpha_2 = -3 \\ \alpha_3 = -4 \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}}$$

Questão 6 : forma de Jordan

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 4 & -1 \\ 1 & 1 & 2 \end{bmatrix} \qquad \Delta(\lambda) = (\lambda-3)^3$$

$$M = (A - \lambda I) = (A - 3I) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

rank = 1

ν = nº colunas − rank = 2

2 blocos de Jordan

$M^2 = \mathbb{O} \rightarrow k$ = maxima dimensão de um dos blocos

logo

$$\operatorname{diag}(J_2(3), J_1(3)) = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Questão 7: $v(t) = ?$

$$\dot{v} = \underbrace{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}}_{A} v \qquad v(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\rightarrow v(t) = e^{At} v(0)$$

$$\rightarrow e^{At} = \alpha(0) I + \alpha_1 A$$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda \rightarrow \begin{matrix} e^{\lambda_1 t} = \alpha_0 + \lambda_1 \alpha_1 \\ e^{\lambda_2 t} = \alpha_0 + \lambda_2 \alpha_1 \end{matrix}$$

Autovalores

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & -1 \\ 6 & \lambda+5 \end{vmatrix} = \lambda^2 + 5\lambda + 6$$

$$(\lambda+3)(\lambda+2)$$

$$\lambda = -2 \qquad \lambda = -3$$

$$\begin{aligned} & e^{-2t} = \alpha_0 - 2\alpha_1 \\ & -(e^{-3t} = \alpha_0 - 3\alpha_1) \end{aligned}$$

$$\boxed{e^{-2t} - e^{-3t} = \alpha_1} \rightarrow \alpha_0 = e^{-2t} + 2\alpha_1$$

$$\boxed{\alpha_0 = 3e^{-2t} - 2e^{-3t}}$$

$$e^{At} = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & 0 \\ 0 & 3e^{-2t} - 2e^{-3t} \end{bmatrix} + \left(e^{-2t} - e^{-3t}\right) \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6e^{-2t} + 6e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix}$$

$$v(t) = e^{At} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\boxed{v(t) = \begin{bmatrix} 2e^{-2t} - e^{-3t} \\ -4e^{-2t} + 3e^{-3t} \end{bmatrix}}$$

Questão 8:

a) $\hat{A} = ?$

$$A = \begin{bmatrix} 8 & -6 \\ 15 & -11 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-8 & 6 \\ -15 & \lambda+11 \end{vmatrix}$$

$$\stackrel{!}{=} \lambda^2 + 3\lambda + 2$$

$$\stackrel{!}{=} (\lambda+1)(\lambda+2)$$

$\lambda = -1 \qquad \lambda = -2$

$$\hat{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

b)

$$Q = \begin{array}{cc} v_1(\lambda_1) & v_2(\lambda_2) \\ \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \end{array} \Rightarrow \boxed{Q = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}}$$

$v_1(\lambda_1) \rightarrow$ para $\lambda_1 = -1$

$$(A - \lambda I)v_1 = \emptyset$$

$$(A + I)v_1 = \emptyset$$

$$\begin{bmatrix} 9 & -6 \\ 15 & -10 \end{bmatrix} \begin{vmatrix} v_{11} \\ v_{21} \end{vmatrix} = \emptyset$$

$3v_{11} - 2v_{21} = \emptyset \leftarrow 9v_{11} - 6v_{21} = \emptyset$

$V_{21} = 3 \quad V_{11} = 2$

$v_2(\lambda_2)$ para $\lambda_2 = -2$

$$(A + 2I)v_2 = \emptyset$$

$$\begin{vmatrix} 10 & -6 \\ 15 & -9 \end{vmatrix} \begin{vmatrix} V_{12} \\ V_{22} \end{vmatrix} = \emptyset$$

$5V_{12} - 3V_{22} = \emptyset$

$V_{22} = 5 \quad V_{12} = 3$

Questão 9:

$\dot{\bar{v}} = \bar{A}\bar{v}$ $\quad \bar{v}(0) = \bar{v}_0$ $\quad y = \bar{C}\bar{v}$

$y(t) = (t+2)\,\text{sen}(t)$

$\lambda = \pm j \rightarrow$ Mult 2.

$$\bar{A} = \begin{bmatrix} \sigma & -\beta & 1 & 0 \\ \beta & \sigma & 0 & 1 \\ 0 & 0 & \sigma & -\beta \\ 0 & 0 & \beta & \sigma \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$e^{\bar{A}t} = \begin{bmatrix} \cos t & -\text{sen}\,t & t\cos t & -t\,\text{sen}\,t \\ \text{sen}\,t & \cos t & t\,\text{sen}\,t & t\cos t \\ 0 & 0 & \cos t & -\text{sen}\,t \\ 0 & 0 & \text{sen}\,t & \cos t \end{bmatrix}$$

$$\bar{v} = e^{\bar{A}t}\bar{v}_0 \Rightarrow \bar{v}_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \bar{C}\bar{v} \rightarrow \bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Questão 10: $h(t) = ?$ $\quad v(0) = 0$

$$\dot{v} = \begin{bmatrix} 0 & 1 \\ \underbrace{-13}_{-\alpha_0} & \underbrace{-4}_{-\alpha_1} \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x$$

$$y = \begin{bmatrix} \underbrace{23}_{\beta_0} & \underbrace{4}_{\beta_1} \end{bmatrix} v$$

$$H(s) = \frac{4s+23}{s^2+4s+13} = \frac{\beta_1 s + \beta_0}{s^2+\alpha_1 s+\alpha_0}$$

$$= \frac{4(s+2) + 15.3}{(s^{\cancel{2}}+2)^2+9} = \frac{4.(s+2)}{(s+2)^2+3^2} + \frac{5.\ 3}{(s+2)^2+3^2}$$

$$\boxed{h(t) = \left(4e^{-2t}\cos(3t) + 5e^{-2t}\sin(2t)\right)u(t)}$$

Lembrando que

$$\mathcal{L}\{e^{-at}x(t)\} = X(s+a)$$

$$\mathcal{L}\{\cos(\beta t)u(t)\} = \frac{s}{s^2+\beta^2}$$

$$\mathcal{L}\{\sin(\beta t)u(t)\} = \frac{\beta}{s^2+\beta^2}$$