EA616 – PR2 (2016):

1 QUESTÃO:

a) → Pontos de equilíbrio

Para $x = 1$ → $f_1 = \dot{v}_1 = v_1 v_2 - 2 v_2 = 0$ ⇒ $\boxed{\begin{matrix}(0,0)\\(2,3)\end{matrix}}$

$\boxed{f(\bar{v}, \bar{x}) = 0}$ ↗ $f_2 = \dot{v}_2 = -3 v_1 + v_2 v_1 = 0$

b) → O sistema linearizado

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial v_1} & \frac{\partial f_1}{\partial v_2} \\ \frac{\partial f_2}{\partial v_1} & \frac{\partial f_2}{\partial v_2} \end{bmatrix} = \begin{bmatrix} v_2 & v_1 - 2 \\ -3 + v_2 & v_1 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial x} \\ \frac{\partial f_2}{\partial x} \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$(0,0)$: $A = \begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$, $(2,3)$: $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

2 QUESTÃO:

$(0,0)$: $A = \begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$; $\Delta(\lambda) = \lambda^2 - 6 = 0$ ⇒ $\lambda_1 = \sqrt{6}$, $\lambda_2 = -\sqrt{6}$; INSTÁVEL (Autovalor positivo)

$(2,3)$: $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, $\Delta(\lambda) = \lambda^2 - 5\lambda + 6 = 0$ ⇒ $\lambda_1 = 3$, $\lambda_2 = 2$; INSTÁVEL (Autovalores positivos)

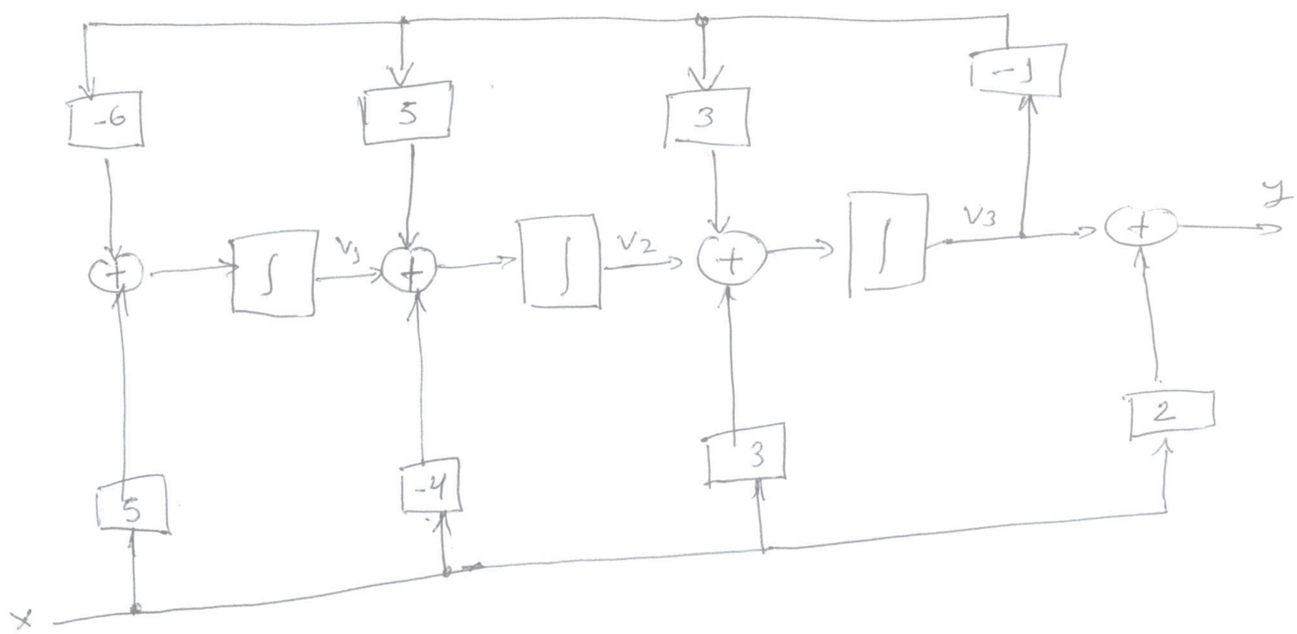
③ QUESTÃO:

$$(p^3+3p^2+5p-6)y(t) = (2p^3+9p^2+6p-7)x(t), \quad p=\frac{d}{dt}$$

$\beta_3 = 2$, $\bar{N}(p) = 3p^2-4p+5$

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -5 \\ 0 & 1 & -3 \end{bmatrix}, \quad c = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}', \quad b = [0 \ 0 \ 1]'$$

$d = 2$



(4) QUESTÃO:

Sistema linear

$$\dot{v} = \underbrace{\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}}_{A} v \quad , \quad v(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C} v$$

a) $Y(s) = \mathcal{L}\{y(t)\}$?

$$Y(s) = C\,(sI - A)^{-1} v(0) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s-2 & 3 \\ -3 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \underbrace{\frac{3(s-2)}{(s-2)^2+9} + \frac{3}{(s-2)^2+9}}$$

b) $$\mathcal{L}^{-1}\{Y(s)\} = y(t) = \boxed{\left(3\exp(2t)\cos(3t) + \exp(2t)\,\text{sen}(3t)\right) u(t)}$$

(5) QUESTÃO:

$$A = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 2 & 7 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow$$

EQUAÇÃO CARACTERÍSTICA

$$\Delta(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6$$

CAYLEY-HAMILTON

$$\Delta(A) = 0$$

$$\therefore \quad A^3 - 6A^2 + 11A - 6I = 0$$

$$\boxed{A^3 - 6A^2 + 11A = 6I}$$

⑥ QUESTÃO: Determine a ~~for~~ forma de Jordan da Matriz

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}, \quad \Delta(\lambda) = \det(\lambda I - A) = \lambda^3$$

$$\lambda = 0$$

P1) $$A - \lambda I = M = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \quad L_1 + L_2$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{matrix} \left.\right\} 2 = \nu(M) = \text{nº blocos de Jordan} \\ \left.\right\} 1 = \text{rank}(M) \end{matrix}$$

P2) $M^2 = 0$ $\quad k = 2$ DIMENSÃO DO MAIOR BLOCO DE JORDAN

$$\therefore \hat{A} = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right]$$

⑦ QUESTÃO:

$$\nu(t) = P^{-1} \bar{\nu}(t), \quad \bar{\nu}(0) = P \nu(0)$$

$$\bar{\nu}(t) = \exp(PAP^{-1}) \underbrace{\bar{\nu}(0)}_{} = \begin{bmatrix} \exp(t) & 0 \\ 0 & \exp(2t) \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4\exp(t) \\ \exp(2t) \end{bmatrix}$$

$$\nu(t) = P^{-1} \bar{\nu}(t) = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4\exp(t) \\ \exp(2t) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -4\exp(t) + 5\exp(2t) \\ \exp(2t) \end{bmatrix}}$$

③ QUESTÃO:

$$A = \begin{bmatrix} -1 & -1 \\ 6 & 4 \end{bmatrix} \Rightarrow \det(\lambda I - A) = \begin{vmatrix} \lambda+1 & 1 \\ -6 & \lambda-4 \end{vmatrix}$$

$$= \lambda^2 - 3\lambda + 2 = 0$$

$\lambda_1 = 1$
$\lambda_2 = 2$

$$M_1 = A - 1I = \begin{bmatrix} -2 & -1 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}$$

$\nu(M_1) = 1 \longrightarrow$
$\text{rank}(M_1) = 1$

$$M_2 = A - 2I = \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$\nu(M_2) = 1 \longrightarrow$
$\text{rank}(M_2) = 1$

$$\boxed{\hat{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}$$

b) $\hat{A} = Q^{-1} A Q$

$(A - I)\, v_1 = 0$

$$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$b = -2a \quad \Rightarrow v_1 = [a \ \ -2a]'$

$(A - 2I)\, v_2 = 0$

$$\begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = 0$$

$d = -3c \quad \Rightarrow v_2 = [c \ \ -3c]'$

$$\boxed{Q_{\text{Geral}} = \begin{bmatrix} a & c \\ -2a & -3c \end{bmatrix}} \Rightarrow Q = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

9) QUESTÃO:

$\dot{v} = \bar{A}\bar{v}$, $\bar{v}(0) = \bar{v}_0$, $y = \bar{C}\bar{v}$

Saída: $y(t) = 10t \exp(t) \operatorname{sen}(2t)$

$\lambda = 2j$ -> MULTIPLICIDADE 2

POLOS CONJUGADOS

$$y_{(t)} = [\, c_1 \quad c_2 \quad c_3 \quad c_4 \,]\, e^t \begin{bmatrix} \cos 2t & -\operatorname{sen} 2t & t\cos 2t & -t\operatorname{sen} 2t \\ \operatorname{sen} 2t & \cos 2t & t\operatorname{sen} 2t & t\cos 2t \\ 0 & 0 & \cos 2t & -\operatorname{sen} 2t \\ 0 & 0 & \operatorname{sen} 2t & \cos 2t \end{bmatrix} \begin{bmatrix} v_{01} \\ v_{02} \\ v_{03} \\ v_{04} \end{bmatrix}$$

$c_1 \Downarrow 0$, $c_2 \Downarrow 10$, $c_3 \Downarrow 0$, $c_4 \Downarrow 0$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\bar{A} = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & 1 \end{bmatrix}}$$

10)

$$Y(s) = \cancel{C(sI-A)^{-1} v_0} + (C(sI-A)^{-1} b + d)\, X(s)$$

$v_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$Y(s) = (C(sI-A)^{-1} b + d)\, X(s)$$

$X(s) \downarrow 1/s$ => função degrau

$$Y(s) = \frac{18s^2 + 86s + 80}{(s+2)(s+4)}$$

$$\boxed{y_u(t) = (10 + 3\exp(-4t) + 5\exp(-2t))\, u(t)}$$