PR2-2015 EA616 David Andrade

Q1.
a) Ptos de Equilíbrio: $f(v) = \dot{v} = 0$

$\dot{v} = 0 \nearrow v = 1$
$\searrow v = -1$

b) • Jacobiano: $\frac{\partial f}{\partial v} = 2v$ • Linearização: $\dot{v} = \frac{\partial f}{\partial v} \cdot v$

$\dot{v} = [2v] v$

• Pto $v = 1$: $\dot{v} = 2v \Rightarrow$ Instável

• Pto $v = -1$: $\dot{v} = -2v \Rightarrow$ Assintóticamente Estável

Q2. Fazendo o Jacobiano:

$\frac{\partial f_1}{\partial v_1} = 2 v_1 v_2 + (v_2^2 - 4)$ $\frac{\partial f_2}{\partial v_1} = (3v_1^2 - 1) v_2 + (v_2^2 - 4) v_2$

$\frac{\partial f_1}{\partial v_2} = (v_1^2 - 1) + 2 v_2 v_1$ $\frac{\partial f_2}{\partial v_2} = (v_1^2 - 1) v_1 + (3v_2^2 - 4) v_1$

$\frac{\partial f_1}{\partial x} = 3$ $\frac{\partial f_2}{\partial x} = 4x$

Em $v_1 = -1$, $v_2 = 2$ $x = 0$:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial v_1} & \frac{\partial f_1}{\partial v_2} \\ \frac{\partial f_2}{\partial v_1} & \frac{\partial f_2}{\partial v_2} \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ 4 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Q2 (Cont...) $\det(A-\lambda I) = \begin{vmatrix} -4-\lambda & -4 \\ 4 & -8-\lambda \end{vmatrix}$

$\det(A-\lambda I) = (4+\lambda)(8+\lambda) - 16$

$\det(A-\lambda I) = \lambda^2 + 12\lambda + 32 - 16 = \underbrace{\lambda^2 + 12\lambda + 16}_{\text{Parte real negativa.}}$

Q3. Representação de variáveis de estado

$\cdot A = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}}_{\text{Forma companheira}}$ $\cdot B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\cdot C = [\bar{\beta}_0 \ \bar{\beta}_1 \ \bar{\beta}_2]$

$\cdot d = [\beta_3]$

Eq. Dif.: $(\alpha_3 p^3 + \alpha_2 p^2 + \alpha_1 p + \alpha_0) y(t) = (\beta_3 p^3 + \beta_2 p^2 + \beta_1 p + \beta_0) x(t)$

$\alpha_3 \downarrow 1$, $\alpha_2 \downarrow 10$, $\alpha_1 \downarrow 9$, $\alpha_0 \downarrow 8$, $\beta_3 \downarrow 2$, $\beta_2 \downarrow 22$, $\beta_1 \downarrow 21$, $\beta_0 \downarrow 17$

$\cdot A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -9 & -10 \end{bmatrix}$ $\cdot B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\cdot \bar{\beta}_0 = \beta_0 - \beta_3 \cdot \alpha_0$ $\bar{\beta}_2 = 2$

$\bar{\beta}_0 = 17 - 2\cdot(8) = 1$

$\bar{\beta}_1 = 21 - 2\cdot(9) = 3$

$\hookrightarrow$ Slide 43 (Cap 17)

$\cdot C = [1 \ 3 \ 2]$ $\cdot d = [2]$

Q4. Primeiramente $(sI-A)^{-1}$:

$(sI-A) = \begin{bmatrix} s+7 & 12 \\ -1 & s \end{bmatrix} \Rightarrow (sI-A)^{-1} = \frac{1}{\det(sI-A)} \cdot \begin{bmatrix} s & -12 \\ 1 & s+7 \end{bmatrix}$

$(sI-A)^{-1} = \frac{1}{s^2+7s+12} \begin{bmatrix} s & -12 \\ 1 & s+7 \end{bmatrix}$

Q4 (Cont).

$$Y(s) = C(sI-A)^{-1}v(0)$$

$$Y(s) = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \underbrace{\frac{1}{s^2+7s+12}}_{\text{Escalar}} \cdot \begin{bmatrix} s & -12 \\ 1 & s+7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Y(s) = \left(\frac{1}{s^2+7s+12}\right) \cdot \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s-12 \\ s+8 \end{bmatrix} \Rightarrow Y(s) = \frac{2s-4}{s^2+7s+12}$$

b) · Utilizando Frações Parciais: $(s^2+7s+12) = (s+3)(s+4)$

$$Y(s) = \frac{2s-4}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4}, \quad A = Y(s)(s+3)\Big|_{s=-3}$$

$$A = -10$$

$$Y(s) = \frac{-10}{s+3} + \frac{12}{s+4} \qquad B = Y(s) \cdot (s+4)\Big|_{s=-4} = +12$$

· Pela inversa: $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[-10 \cdot e^{-3t} + 12 \cdot e^{-4t}\right] u(t)$

Q5. · Por Cayley - Hamilton (Slide 16, Cap 18)

· Reescrevendo: $A^4 + 4A^2 = \underbrace{-(5A^3+3A+2I)}$

$$A^4 + 5A^3 + 4A^2 + 3A + 2I = 0$$

↳ Encontramos uma equação de A, que deve servir também como polinômio característico:

$$\det(sI-A) = \Delta(A) = 0 \Rightarrow \Delta(\lambda) = \lambda^4 + 5\lambda^3 + 4\lambda^2 + 3\lambda + 2 = 0$$

Q6. Procedimento descrito no Slide 67, Cap 18:

$$M_1 = (A - (1) \cdot I) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \underbrace{\text{Linha } 2 = -\text{Linha } 1}$$

$$\nu(M_1) = 3 - \overbrace{\text{rank}(M_1)}^{2}$$

$$\nu(M_1) = 3 - 2 = 1$$

• Logo, temos apenas um bloco de Jordan!!

$$J_3(1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Bloco de Jordan $1 \mp j$

Q7. $$\dot{v} = \underbrace{\begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}}_{A} v$$

Bloco de Jordan $1 \mp j$

• Conforme exemplo 1.16 (Slide 55 Cap 18):

$$e^{At} = e^t \cdot \begin{bmatrix} \cos t & -\text{sen}\, t & t\cos t & -t\,\text{sen}\, t \\ \text{sen}\, t & \cos t & t\,\text{sen}\, t & t\cos t \\ 0 & 0 & \cos t & -\text{sen}\, t \\ 0 & 0 & \text{sen}\, t & \cos t \end{bmatrix}$$

Apenas coluna 3

$$v(t) = e^{At} \cdot v(0) = e^t \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = e^t \cdot \begin{bmatrix} t\cos t \\ t\,\text{sen}\, t \\ \cos t \\ \text{sen}\, t \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} v(t)$$

$$y(t) = t\,\text{sen}\, t\, e^t$$

Q8. $\cdot \det(A-\lambda I) = \begin{vmatrix} 5-\lambda & 1 \\ -4 & 1-\lambda \end{vmatrix} = 5 - 6\lambda + \lambda^2 + 4$

a)

$\det(A-\lambda I) = \lambda^2 - 6\lambda + 9 = (\lambda-3)^2$ $\rightarrow$ Auto valor 3 com mult. 2

$M_3 = (A - 3 \cdot I) = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, $L_2 = L_1 \cdot (-2)$, $\nu(M_3) = 2 - 1 = 1$

$\hookrightarrow$ Um bloco de Jordan!

$\hat{A} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

b) Calculando Q: $M_3 \overbrace{\begin{bmatrix} a \\ b \end{bmatrix}}^{v_1} = 0 \Rightarrow \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$

(Slide 76 Cap18)

$2a + b = 0 \Rightarrow b = -2a$ $\qquad v_1 = \begin{bmatrix} a \\ -2a \end{bmatrix}$

$M_3 v_2 = v_1$:

$\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a \\ -2a \end{bmatrix} \Rightarrow 2c + d = a \Rightarrow d = a - 2c$

Assim: $Q = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a & c \\ -2a & a-2c \end{bmatrix}$

Fazendo $a = 1$ e $c = 1$: $Q = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$

$$\text{Modo } e^{\pm jt} \text{ mult. } 1$$

Q9 Analizando: $y(t) = \underbrace{t}_{\text{Modo } e^{0t} \text{ com mult } 2.} + \widetilde{\cos(3t)}$

$$\overset{\text{Jordan}}{\bar{A}} = \begin{bmatrix} 0 & -3 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow e^{\bar{A}t} = \begin{bmatrix} \cos 3t & -\operatorname{sen} 3t & 0 & 0 \\ \operatorname{sen} 3t & \cos 3t & 0 & 0 \\ 0 & 0 & e^0 & t \cdot e^0 \\ 0 & 0 & 0 & e^0 \end{bmatrix}$$

$v(t) = e^{\bar{A}t} \cdot \bar{v}(0) \Rightarrow$ Temos que "selecionar" as colunas 1 e 4

$$v(t) = \begin{bmatrix} \cos 3t & -\operatorname{sen} 3t & 0 & 0 \\ \operatorname{sen} 3t & \cos 3t & 0 & 0 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 3t \\ \operatorname{sen} 3t \\ t \\ 1 \end{bmatrix}$$ Selecionar

$$y(t) = \bar{C} \cdot v(t) = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos 3t \\ \operatorname{sen} 3t \\ t \\ 1 \end{bmatrix} = \cos 3t + t$$ //

Q10. · Aplicando a transformação p/ função transferência:

$$\cdot (sI - A) = \begin{bmatrix} s+3 & 2 \\ -1 & s \end{bmatrix} \Rightarrow (sI-A)^{-1} = \frac{1}{s(s+3)+2} \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^2+3s+2} \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix} \Rightarrow H(s) = \frac{1}{(s+2)(s+1)} \overbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}^{c} \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix} \overbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}^{b}$$

$$H(s) = \frac{1}{(s+2)(s+1)} \cdot \begin{bmatrix} s+1 & s+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow H(s) = \frac{2\cancel{(s+1)}}{\cancel{(s+1)}(s+2)} = \frac{2}{(s+2)}$$

$$Y_n(s) = H(s) \cdot R(s) = \frac{2}{(s+2) s^2} \rightsquigarrow Y_n(s) = \frac{1}{s^2} - \frac{1}{2 \cdot s} + \frac{1}{2} \cdot \frac{1}{s+2}$$ (Frações Parciais)

$$y_n(t) = \mathcal{L}^{-1}\{Y_n(s)\} = \left(t - \frac{1}{2} + \frac{e^{-2t}}{2}\right) u(t)$$ //