EA 616 - Análise Linear de Sistemas - 2S 16 PR1 ( )

1) QUESTÃO

a) Função de transferência (condições iniciais nulas)

$$\ddot{y} + 12y = \ddot{x} + 16x$$

-> LAPLACE

$$\mathcal{L}\{\ddot{y}\} + 12\,\mathcal{L}\{y\} = \mathcal{L}\{\ddot{x}\} + 16\,\mathcal{L}\{x\}$$

$$s^2 Y(s) + 12\,Y(s) = s^2 X(s) + 16\,X(s)$$

$$Y(s)\,(s^2+12) = X(s)\,(s^2+16)$$

F.T -> $$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+16}{s^2+12}}$$

b)

$$x(t) = \operatorname{sen}(3t) = \frac{1}{2j}e^{3j} - \frac{e^{-3j}}{2j}$$

Modo forçado $\lambda = 3j$

Sistema LTI:

$$y_f(t) = H(3j)\,\operatorname{sen}(3t)$$

$$y_f(t) = \left(\frac{s^2+16}{s^2+12}\right)\Bigg|_{s=3j} * \operatorname{sen}(3t)$$

$$\therefore \boxed{y_f(t) = \frac{7}{3}\operatorname{sen}(3t)}$$

2) QUESTÃO:

$$(p^2+9)(p+2)y = (7p^2+4p+45)x \,, \quad \left(\begin{array}{c}\text{condições}\\ \text{iniciais}\\ \text{nulas}\end{array}\right) \quad \begin{array}{c}\text{sistema}\\ \text{LTI}\end{array}$$

$$\left.\frac{N(p)}{D(p)}\right|_{p=s} = H(s) = \frac{N(s)}{D(s)} = \frac{7s^2+4s+45}{(s^2+9)(s+2)}$$

$$\frac{7s^2+4s+45}{(s^2+9)(s+2)} = \frac{As}{(s^2+9)} + \frac{C}{(s+2)}$$

$$7s^2+4s+45 = (A+C)s^2 + 2As + 9C$$

$$A = 2 \,, \quad C = 5$$

$$\rightarrow H(s) = \frac{2s}{(s^2+9)} + \frac{5}{(s+2)}$$

(Transformadas inversa de Laplace) $\mathcal{L}^{-1}\{\;\}$

$$\boxed{h(t) = \left(5\exp(-2t) + 2\cos(3t)\right)u(t)}$$

3) QUESTÃO:

a) $\mathcal{L}\{\ddot{y} + 2\dot{y} - 15y\} = 0 \quad y(0),\ \dot{y}(0)$

$$s^2Y(s) - s\,y(0) - \dot{y}(0) + 2s\,Y(s) - 2y(0) - 15Y(s) = 0$$

$$\boxed{Y(s) = \frac{(s+2)y(0) + \dot{y}(0)}{(s-3)(s+5)}}$$

b) $y(t)$? $\quad y(0) = 2,\ \dot{y}(0) = 30$

$$Y(s) = \frac{2s+34}{(s-3)(s+5)}$$

$$Y(s) = \frac{5}{(s-3)} - \frac{3}{(s+5)}$$

$$\boxed{y(t) = \left(5\exp(3t) - 3\exp(-5t)\right)u(t)}$$

④ QUESTÃO:

SLIT (DA FIGURA)

$$\begin{cases} s = -1 & \text{zero} \\ s = -10^3 & \text{polo} \end{cases}$$

$$H(s) = K\,\frac{(s+1)}{(s+10^3)}$$

$$H(j\omega) = K\,\frac{(j\omega+1)}{(j\omega+10^3)}$$

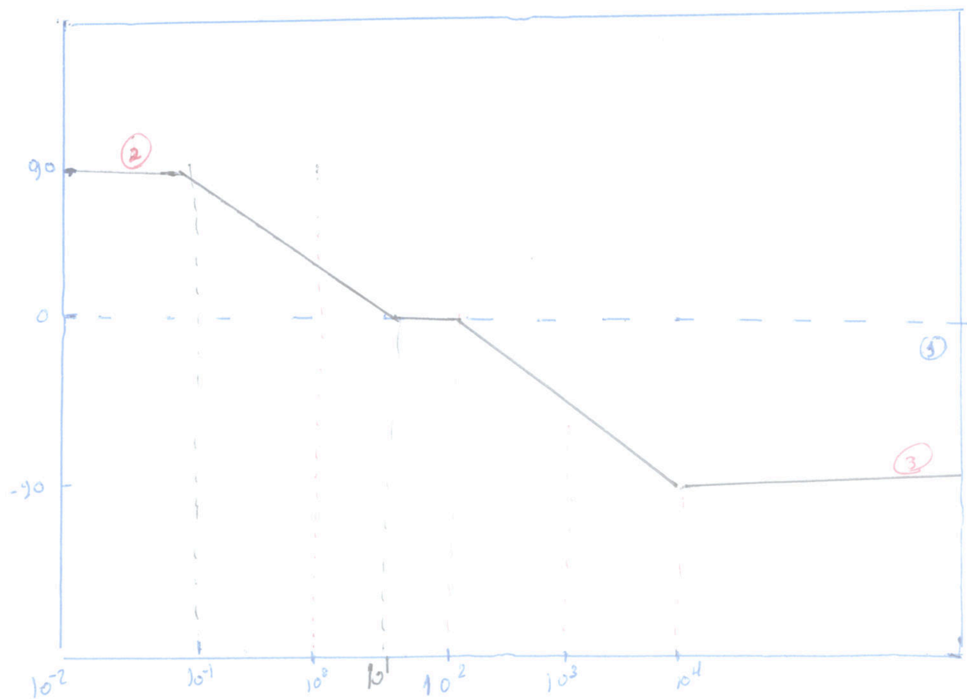
MÓDULO: $20\log|H(j\omega)| = \underbrace{20\log K}_{1} + \underbrace{20\log|j\omega+1|}_{2} - \underbrace{20\log|j\omega+10^3|}_{3}$

A parte ① do módulo é a reta constante

$20\log K = 20$

$\boxed{K = 10}$ $\Rightarrow H(s) = 10\,\frac{(s+1)}{(s+10^3)}$

a) FASE: $\phi[H(j\omega)] = \underbrace{\phi(10)}_{1} + \underbrace{\phi(j\omega+1)}_{2} - \underbrace{\phi(j\omega+10^3)}_{3}$



b) $X(t) = \cos(1000t)$

$s = j\omega = 1000j$

$H(j\omega) = \frac{K(j\omega + 1)}{(j\omega + 1000)}$

Da figure:

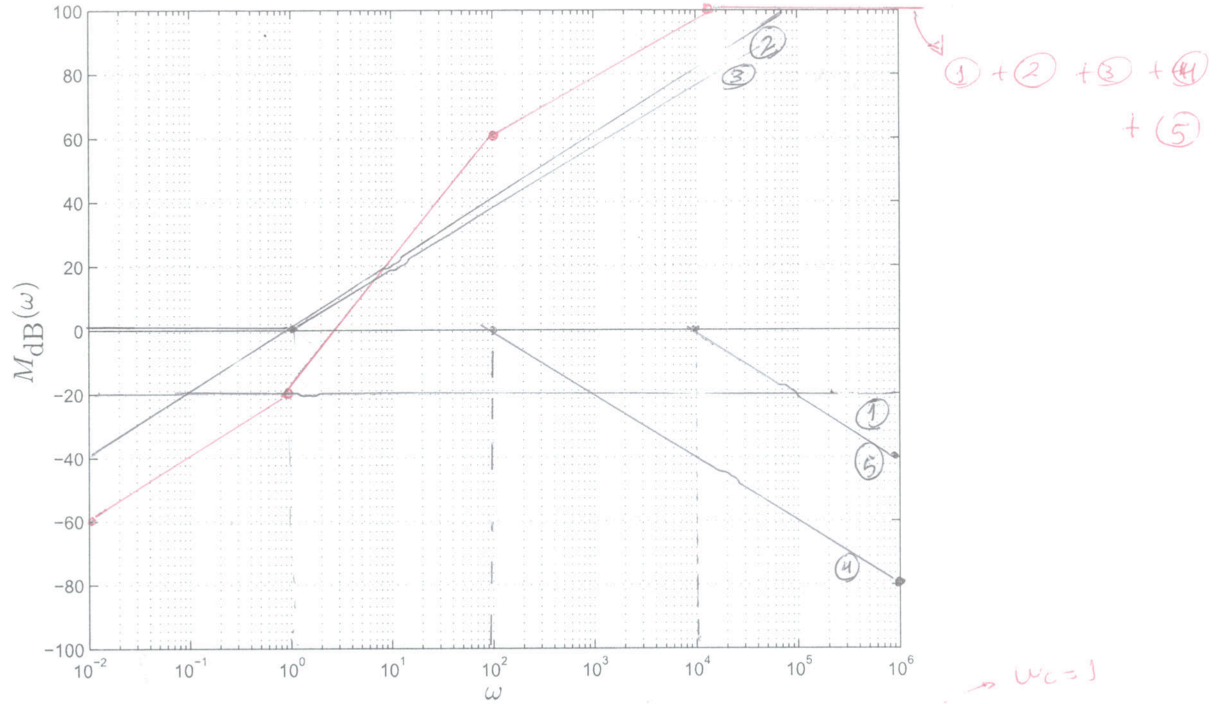
$y_f(t) = \underbrace{H(1000j)}_{..} \cdot \cos(1000t)$

$y_f(t) = \underbrace{10}_{} \cos(1000t - 45°)$

5ª Questão: a) Esboce as assíntotas do módulo (diagrama de Bode em escala logarítmica) do sistema linear invariante no tempo descrito pela função de transferência

$$H(s) = \frac{s(s+1)10^5}{(s+100)(s+10000)}$$

$$H(j\omega) = \frac{10^{-1}(j\omega)(j\omega+1)}{\left(1+\frac{j\omega}{10^2}\right)\left(1+\frac{j\omega}{10^4}\right)}$$



Módulo:

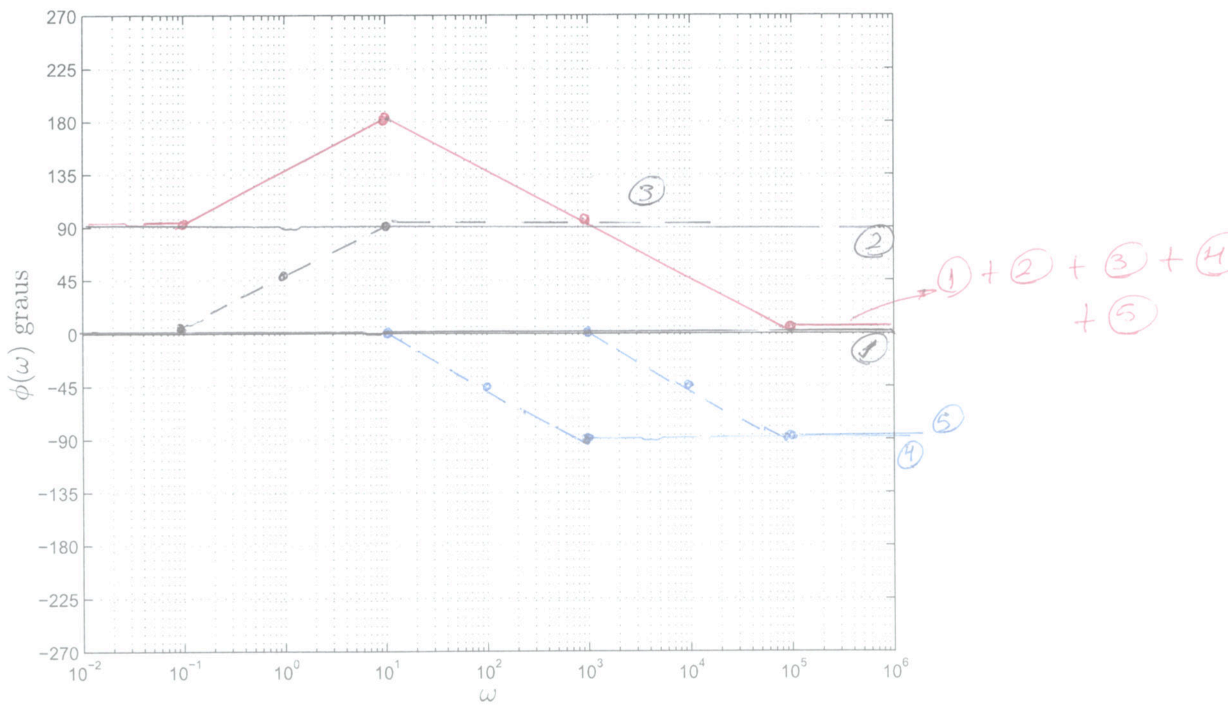
$$20\log|H(j\omega)| = \underbrace{20\log(10^{-1})}_{1} + \underbrace{20\log(j\omega)}_{2} + \underbrace{20\log(j\omega+1)}_{3} - \underbrace{20\log\left(1+\frac{j\omega}{10^2}\right)}_{4} - \underbrace{20\log\left(1+\frac{j\omega}{10^4}\right)}_{5}$$

$\to \omega_c = 1$

$\to \omega_c = 10^2$

$\to \omega_c = 10^4$

b) Esboce as assíntotas da fase (diagrama de Bode em graus) do sistema.



6) QUESTÃO:

* SLIT → Resposta ao impulso:

$$h(t) = t\,\exp(-2t)\,u(t)$$

$$\mathcal{L}\{\ \}$$

$$H(s) = \frac{1}{(s+2)^2}$$

→ Para uma entrada $x(t) = 5\exp(-2t)$

$$\mathcal{L}\{\ \}$$

$$X(s) = \frac{5}{(s+2)}$$

Solução forçada → $Y_f(s) = H(s)\,X(s)$

$$= \frac{5}{(s+3)^3} \rightarrow \mathcal{L}^{-1}\{\ \}$$

Inversa de Laplace

$$\boxed{y_f(t) = \frac{t^2}{2}\,5\exp(-2t)}$$

7) QUESTÃO:

$$y(t) = (t^2 + 5t + 10)\exp(-2t)$$

$$y(t) = \underbrace{t^2\exp(-2t)} + 5t\exp(-2t) + 10\exp(-2t)$$

$\lambda = -2$ MODO TRIPLO → $D(p)\,y = 0$

$$\boxed{(p+2)^3\,y(t) = 0}$$ EDH

* Condições iniciais:

$$y(0) = \left[t^2\exp(-2t) + 5t\exp(-2t) + 10\exp(-2t)\right]\Big|_{t=0}$$

$$= \underline{10}$$

$$\dot{y}(0) = \dot{y}(t)\Big|_{t=0} = \left[-2t^2\exp(-2t) - 8t\exp(-2t) - 15\exp(-2t)\right]\Big|_{t=0}$$

$$= \underline{-15}$$

$$\ddot{y}(0) = \ddot{y}(t)\Big|_{t=0} = \left[4t^2\exp(-2t) + 12t\exp(-2t) + 22\exp(-2t)\right]\Big|_{t=0}$$

$$= \underline{22}$$

9) QUESTÃO :

$$p(p+1)\,y = t - \exp(-t) \quad , \quad y(0) = \dot{y}(0) = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = -1$$

$$x(t) = t\,\exp(0t) - \exp(-t)$$

$$\gamma_1 = 0 \qquad \gamma_3 = -1$$

$$\gamma_2 = 0$$

MODOS FORÇADOS

$$y(t) = \underbrace{a_1 + a_2 \exp(-t)}_{y_H(t)} + \underbrace{b_1 t + b_2 t^2 + b_3 t \exp(-t)}_{y_f(t)}$$

SOLUÇÃO FORÇADA

$$y_f(t) = b_1 t + b_2 t^2 + b_3 t \exp(-t)$$

$$\dot{y}_f(t) = b_1 + 2b_2 t + b_3 \exp(-t) - b_3 t \exp(-t)$$

$$\ddot{y}_f(t) = 2b_2 - 2b_3 \exp(-t) + b_3 t \exp(-t)$$

$$D(p)\,y_f(t) = t - \exp(-t)$$

$$b_1 + 2b_2 + 2b_2 t - b_3 \exp(-t) = t - \exp(-t)$$

$$b_1 = -1 \,, \quad b_2 = \frac{1}{2} \,, \quad b_3 = 1$$

$$y(t) = a_1 + a_2 \exp(-t) + \frac{t^2}{2} - t + t\exp(-t)$$

Condições iniciais

$$y(0) = 0 = a_1 + a_2$$

$$\dot{y}(0) = 0 = a_2$$

$$a_1 = a_2 = 0$$

$$\therefore \quad \boxed{y(t) = \frac{t^2}{2} - t + t\exp(-t)}$$

9) QUESTÃO:

a) $Y(z)$?

$$\mathcal{Z}\{y[n+2] + y[n+1] - 2y[n]\} = 0 \quad , \quad y[0]=1 \quad , \quad y[1]=13$$

$$z^2 Y(z) - z^2 y[0] - z\,y[1] + z\,Y(z) - z\,y[0] - 2Y(z) = 0$$

$$(z^2+z-2)\,Y(z) = z^2 + 14z$$

$$\boxed{Y(z) = \frac{z^2+14z}{(z-1)(z+2)}}$$

b) $y[n] = \mathcal{Z}^{-1}\{Y(z)\}$

$$Y(z) = \frac{z^2+14z}{(z-1)(z+2)} = z\left[\frac{A}{(z-1)} + \frac{B}{(z+2)}\right]$$

$A = 5$

$B = -4$

$$Y(z) = \frac{5z}{(z-1)} - \frac{4z}{(z+2)}$$

$\mathcal{Z}^{-1}\{\}$

$$\therefore \boxed{y[n] = \left(5(1)^n - 4(-2)^n\right)u[n]}$$

10 QUESTÃO:

a) $(P-3)\,y[n] = 12n\,3^n$, $P\,y[n] = y[n+1]$

$\underbrace{(P-3)}_{\lambda = 3}$ $\underbrace{12n\,3^n}$

$\gamma_1 = 3$
$\gamma_2 = 3$ } MODOS FORÇADOS

$$y[n] = \underbrace{a_1 (3)^n}_{y_H[n]} + \underbrace{b_1 n^2 (3)^n + b_2 n (3)^n}_{y_f[n]}$$

$\rightarrow$ $D(P)\,y_f[n] = 12n\,3^n$

$\rightarrow$ $b_1 (n+1)^2 (3)^{n+1} + b_2 (n+1)\,3^{n+1} - 3\,(b_1 n^2 3^n + b_2 n\,3^n) = 12n\,3^n$

$b_1 = 2$, $b_2 = -2$

$$y_f[n] = (-2n + 2n^2)\,3^n$$

b) $y[n]$? para $y[0] = 1$

$$y[n] = a_1 (3)^n - 2n\,3^n + 2n^2\,3^n$$

$\rightarrow$ $y[0] = 1 = a_1$

$\therefore$ $\boxed{y[n] = (1 - 2n + 2n^2)\,3^n}$