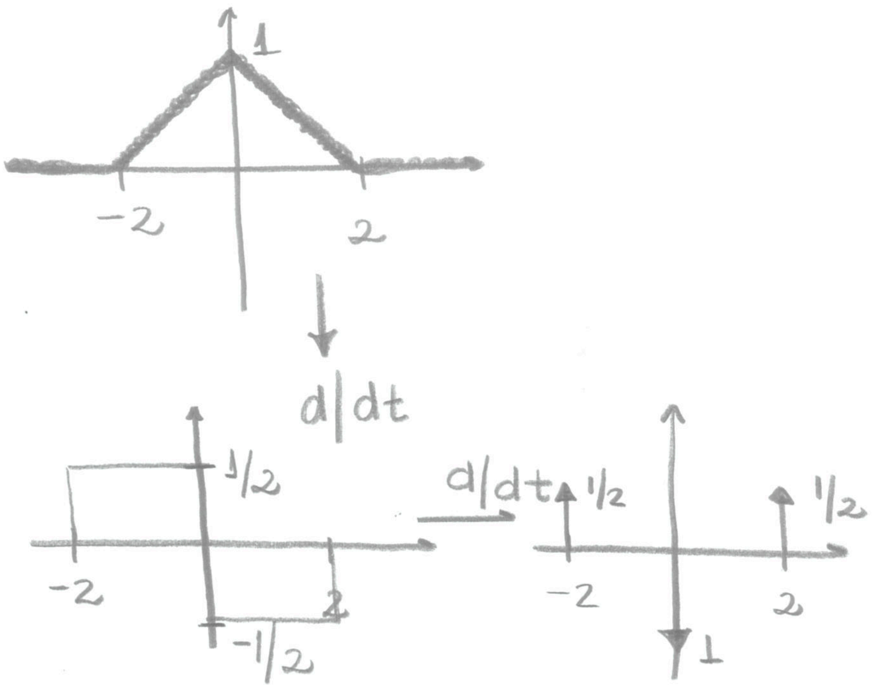
Resolução PR3-2014:

① $\mathcal{F}\{TRI_4(t)\}$



$$\boxed{X(\omega) = \frac{1}{(j\omega)^2}\left(\frac{e^{2j\omega}}{2} - 1 + \frac{e^{-2j\omega}}{2}\right)}$$

② $\mathcal{F}\{\cos(10t)\,G_4(t)\} = \mathcal{F}\left\{\frac{e^{10jt}}{2}G_4(t) + \frac{e^{-10jt}}{2}G_4(t)\right\}$

Propriedades:

$$\mathcal{F}\{x(t)\,e^{j\omega_0 t}\} = X(\omega-\omega_0)$$

$$\mathcal{F}\{G_T(t)\} = T\,Sa(\omega T/2)$$

Então, $\rightarrow \mathcal{F}\{G_4(t)\} = 4\,Sa(2\omega)$

$$\mathcal{F}\{\cos(10t)\,G_4(t)\} = \frac{1}{2}\cdot 4\,Sa(2(\omega-10)) + \frac{1}{2}\cdot 4\,Sa(2(\omega+10))$$

$$= \boxed{2\,Sa(2\omega-20) + 2\,Sa(2\omega+20)}$$

③

$$I = \int_{-\infty}^{\infty} Sa(2t)\, Sa(3t)\, dt$$

<u>Propriedades</u>

$$\int_{-\infty}^{\infty} x(t)\, dt = X(0)$$

$$\mathcal{F}\{x(t)\, y(t)\} = \frac{1}{2\pi} x(t) * y(t)$$

$$\mathcal{F}\{Sa\left(\frac{\omega_0 T}{2}\right)\} = \frac{2\pi}{\omega_0} G_{\omega_0}(\omega)$$

$$I = \mathcal{F}\{Sa(2t)\; Sa(3t)\}\Big|_{\omega=0}$$

$$I = \frac{1}{2\pi}\, \mathcal{F}\{Sa(2t)\} * \mathcal{F}\{Sa(3t)\}\Big|_{\omega=0}$$

$$\mathcal{F}\{Sa(2t)\} = \frac{2\pi}{4} G_4(\omega)$$

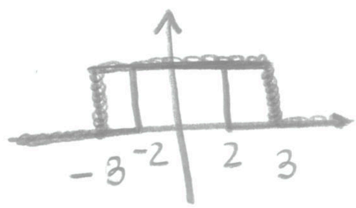
$$\mathcal{F}\{Sa(3t)\} = \frac{2\pi}{6} G_6(\omega)$$

$$I = \frac{1}{2\pi}\left[\left(\frac{2\pi}{4} G_4(\omega)\right) * \left(\frac{2\pi}{6} G_6(\omega)\right)\right]_{\omega=0}$$

Usando a definição de convolução

$$I = \frac{1}{\cancel{2\pi}} \cdot \frac{\cancel{2\pi}}{\cancel{4}} \cdot \frac{2\pi}{6}\left[\int_{-\infty}^{\infty} G_4(\beta) \cdot G_6(\omega-\beta)\, d\beta\right]_{\omega=0}$$

$$I = \frac{\pi}{12}\int_{-\infty}^{\infty} G_4(\beta) \cdot G_6(-\beta)\, d\beta$$



$$I = \frac{\pi}{12} \cdot 2 \cdot \int_{-2}^{0} d\beta = \frac{\pi}{12} \cdot 2 \cdot 2 = \boxed{\pi/3}$$

(4) $X(\omega) = G_2(\omega+1)$

$\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$

$$\mathcal{F}\{X(t)\} = \frac{e^{2j\omega} - 1}{j\omega} = 2\pi x(-\omega)$$

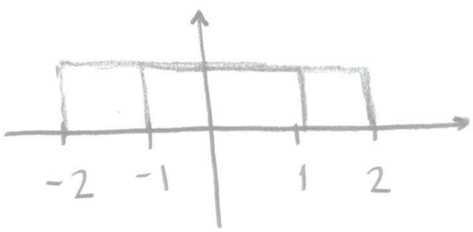
$$x(-\omega) = \frac{e^{2j\omega} - 1}{2\pi j\omega}$$

$\downarrow$

$$x(t) = \frac{e^{-2jt} - 1}{2\pi j(-t)} \longrightarrow \boxed{x(t) = \frac{1 - e^{-2jt}}{2\pi j t}}$$

(5) $x(t) = Sa(t)\, Sa(2t)$

a) $X(\omega) = \left(\frac{2\pi}{2} G_2(\omega)\right) * \left(\frac{2\pi}{4} G_4(t)\right) \cdot \frac{1}{2\pi}$



$\omega_{max} = \frac{4+2}{2} = 3$

$B = \frac{\omega_{max}}{2\pi} = \frac{3}{2\pi}$

$T < \frac{1}{2B} \rightarrow \boxed{T < \frac{\pi}{3}}$

b) $x_a(t) = \sum x(k)\,\delta(t-k) \rightarrow T=1 \rightarrow \omega_0 = 2\pi$

$$H(j\omega) = \frac{T \cdot G_{\omega_0}(\omega)}{P(\omega)}$$

$$P(\omega) = \mathcal{F}\{\delta(t)\} = 1$$

$$\boxed{H(j\omega) = G_{2\pi}(\omega)}$$

6) $x(t) = t\cos(10t)\,e^{-5t}\,u(t)$

Propiedades

$$\mathcal{L}\{e^{-\alpha t}\cos(\beta t)\,u(t)\} = \frac{s+\alpha}{(s+\alpha)^2+\beta^2}$$

$$\mathcal{L}\{t^m x(t)\} = (-1)^m \frac{d^m X(s)}{ds^m}$$

$$\mathcal{L}\{t\,\underbrace{\cos(10t)\,e^{-5t}\,u(t)}_{x(t)}\} = (-1)\cdot\frac{d\,X(s)}{ds}$$

$$X(s) = \frac{s+5}{(s+5)^2+10^2} = \frac{s+5}{s^2+10s+125}$$

$$-\frac{d}{ds}X(s) = -\frac{\left[(s^2+10s+125) - \overbrace{(s+5)(2s+10)}^{2s^2+20s+50}\right]}{(s^2+10s+125)^2}$$

$$-\frac{d}{ds}X(s) = -\frac{[-s^2-10s+75]}{(s^2+10s+125)^2} \rightarrow \boxed{\mathcal{L}\{x(t)\} = \frac{s^2+10s-75}{(s^2+10s+125)^2}}$$

7 $X(s) = \dfrac{s+15}{\underbrace{s^2-9}_{(s-3)(s+3)}} \quad -3 < Re(s) < 3$

$$X(s) = \frac{s+15}{(s+3)(s-3)} = \frac{A}{(s+3)} + \frac{B}{(s-3)} \qquad A=-2 \quad B=3$$

$$X(s) = \underbrace{\frac{-2}{s+3}}_{Re(s)>-3} + \underbrace{\frac{3}{s-3}}_{Re(s)<3}$$

Propriedades

$$\mathcal{L}\{t^m/m!\, e^{-at}u(t)\} = \frac{1}{(s+a)^{m+1}} \qquad Re(s+a)>0$$

$$\mathcal{L}\{(-t)^m/m!\, e^{at}u(-t)\} = \frac{1}{(-s+a)^{m+1}} \qquad Re(s-a)<0$$

$$\boxed{x(t) = -2e^{-3t}u(t) - 3e^{3t}u(-t)}$$

8 $1 = \int_{-\infty}^{\infty} t h(t)\,dt \qquad \ddot{y}(t) + 5\dot{y}(t) + 6y(t) = x(t-5)$

$x(t) = e^{st}$

$e^{st}[s^2H(s) + 5sH(s) + 6H(s)] = e^{s(t-5)} \qquad y(t) = H(s)e^{st}$

$$H(s) = \frac{e^{-5s}}{s^2+5s+6}$$

Propiedades

$$\int_{-\infty}^{\infty} x(t)\,dt = X(s)\Big|_{s=0}$$

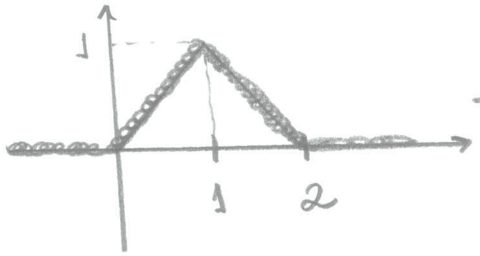
$$\mathcal{L}\{t\,x(t)\} = -\frac{d}{ds}X(s)$$

$$1 = -\frac{d}{ds}H(s)\Big|_{s=0} = -\frac{d}{ds}\left(\frac{e^{-5s}}{s^2+5s+6}\right)\Big|_{s=0}$$

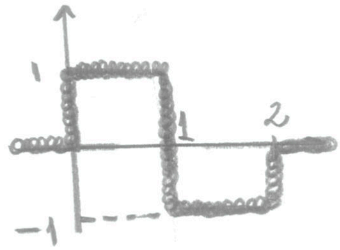
$$1 = -\frac{\left(-5e^{-5s}(s^2+5s+6) - e^{-5s}(2s+5)\right)}{(s^2+5s+6)^2}\Big|_{s=0}$$

$$1 = -\frac{(-5(6)-(5))}{36} = \boxed{\frac{35}{36}}$$

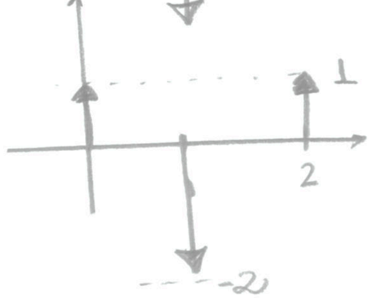
9) $x(t) = TRI_2(t-1) \longrightarrow X(s) = ?$



$\longrightarrow d/ds$



$\downarrow d/ds$



$$\boxed{X(s) = \frac{1-2e^{-s}+e^{-2s}}{s^2} \qquad s=\mathbb{C}}$$

(10) $(p^3 L_1 C_2 L_3 + p^2 2 L_1 C_2 R + p(L_1+L_3) + 2R)\, y = 2R x$

$$H(s) = \frac{1}{D(\lambda)} \quad D(\lambda) = \lambda^3 + 2\lambda^2 + 2\lambda + 1 \quad \lambda = \frac{s}{\omega_c}$$

$$\frac{y}{x} = \frac{2R}{p^3 L_1 C_2 L_3 + p^2 2 L_1 C_2 R + p(L_1+L_3) + 2R}$$

Dividindo por $L_1 C_2 L_3$

$$\frac{y}{x} = \frac{2R / L_1 C_2 L_3}{p^3 + p^2 \frac{2R}{L_3} + p\left(\frac{L_1+L_3}{L_1 C_2 L_3}\right) + \frac{2R}{L_1 C_2 L_3}}$$

$$\lambda = s/\omega_c$$

$$\frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3} = \frac{2R / L_1 C_2 L_3}{p^3 + p^2 \frac{2R}{L_3} + p\left(\frac{L_1+L_3}{L_1 C_2 L_3}\right) + \frac{2R}{L_1 C_2 L_3}}$$

$$\begin{cases} \dfrac{2R}{L_1 C_2 L_3} = \omega_c^3 \\[2ex] \dfrac{2R}{L_3} = 2\omega_c \rightarrow \boxed{L_3 = \dfrac{R}{\omega_c}} \\[2ex] \dfrac{L_1+L_3}{L_1 C_2 L_3} = 2\omega_c^2 \end{cases}$$

$$\rightarrow L_1 + R/\omega_c = 2\omega_c^2 \cdot \frac{2R}{\omega_c^3}$$

$$\boxed{L_1 = \frac{3R}{\omega_c}}$$

$$C_2 = \frac{2R}{\omega_c^3} \cdot \frac{\omega_c}{R} \cdot \frac{\omega_c}{3R} \rightarrow \boxed{C_2 = \frac{2}{3R\omega_c}}$$