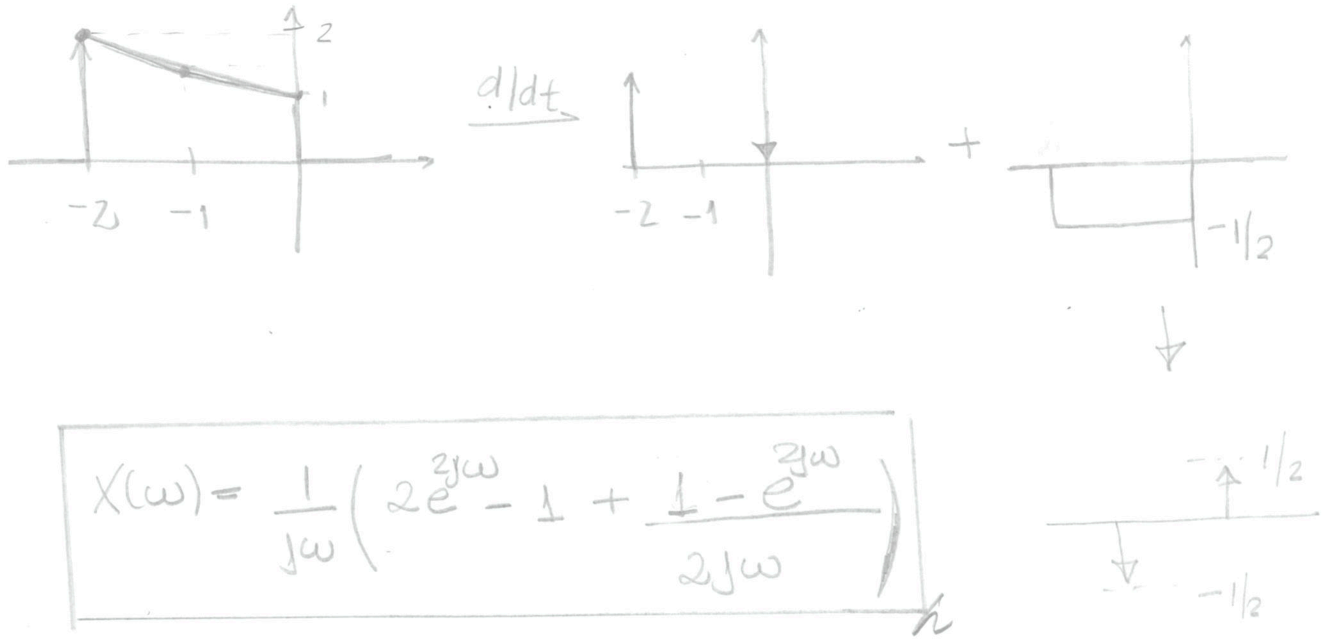
Resolução PR3:

1. $x(t) = (-t/2+1)\, G_2(t+1)$



$$\boxed{X(\omega) = \frac{1}{j\omega}\left(2e^{2j\omega} - 1 + \frac{1-e^{2j\omega}}{2j\omega}\right)}$$

2. $\mathcal{F}\left\{\frac{\exp(-jt)}{t^2+4}\right\}$

$$\mathcal{F}\{e^{-a|t|}\} = \frac{2a}{\omega^2+a^2}$$

$$\mathcal{F}\{x(t-\tau)\} = e^{-j\omega\tau} X(\omega)$$

$$\mathcal{F}\{e^{-2|t-1|}\} = \frac{4e^{-j\omega}}{\omega^2+4}$$

$$\mathcal{F}\left\{\frac{4e^{-jt}}{t^2+4}\right\} = 2\pi \cdot e^{-2|-\omega-1|}$$

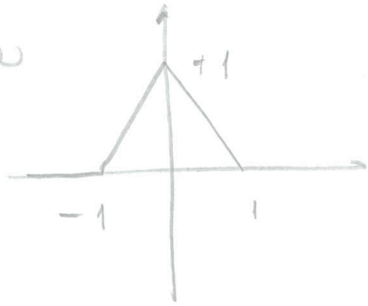
$$\boxed{\mathcal{F}\left\{\frac{e^{-jt}}{t^2+4}\right\} = \frac{\pi}{2} e^{-2|\omega+1|}}$$

③ $I = \int x^2(t)\,dt \qquad x(t) = Sa^2(t/2)$

$$I = \frac{1}{2\pi}\int \left(\mathcal{F}\{Sa^2(t/2)\}\right)^2 d\omega$$

$$I = \frac{1}{2\pi}\int \left(\frac{2\pi}{1}\,TRI_2(\omega)\right)^2 d\omega$$

$$I = 2\pi \int \left(TRI_2(\omega)\right)^2 d\omega$$



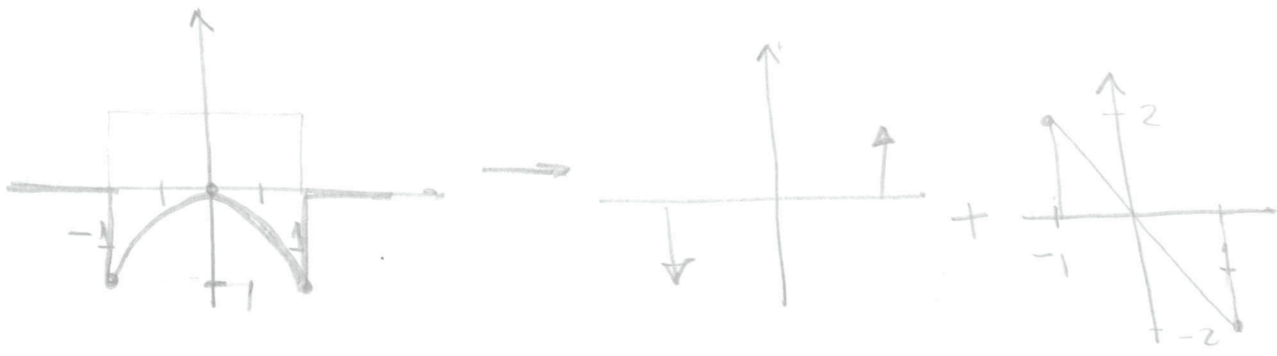
$$I = 2\pi \cdot 2 \cdot \int_{-1}^{0} \underbrace{(t+1)^2}_{t^2+2t+1} d\omega$$

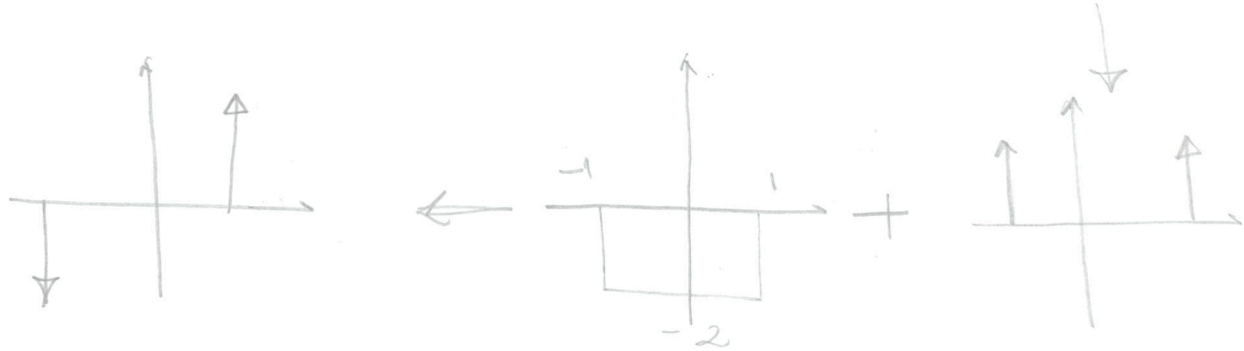
$$I = 4\pi \left(\frac{t^3}{3} + t^2 + t\right)\Bigg|_{-1}^{0}$$

$$4\pi \cdot \left(-\left(-1/3 + \cancel{1} - \cancel{1}\right)\right)$$

$$\boxed{I = 4\pi/3}$$

④ $X(\omega) = -\omega^2 G_2(\omega)$





$$\mathcal{F}\{X(t)\} = \frac{1}{j\omega}\left(e^{-j\omega} - e^{+j\omega} + \frac{2e^{j\omega}+2e^{-j\omega}}{j\omega} + \frac{2e^{-j\omega}-2e^{j\omega}}{(j\omega)^2}\right)$$

$$x(t) = \frac{1}{2\pi(+jt)}\left(-e^{jt} + e^{-jt} + \frac{2e^{-jt}+2e^{jt}}{+jt} + \frac{2e^{jt}-2e^{-jt}}{(+jt)^2}\right)$$

$$\boxed{x(t) = \frac{1}{\pi}\left(\frac{e^{-jt}-e^{jt}}{2jt} - \frac{e^{jt}+e^{-jt}}{t^2} + \frac{e^{jt}-e^{-jt}}{jt^3}\right)}$$

⑤ a) $x(t) = Sa(2t)\cos(10t)$

$$X(\omega) = \frac{1}{2\pi}\mathcal{F}\{Sa(2t)\} * \mathcal{F}\{\cos(10t)\}$$

$$X(\omega) = \frac{1}{2\pi}\left(\frac{2\pi}{4}G_4(\omega)\right) * \left(\pi\delta(\omega-10) + \pi\delta(\omega+10)\right)$$

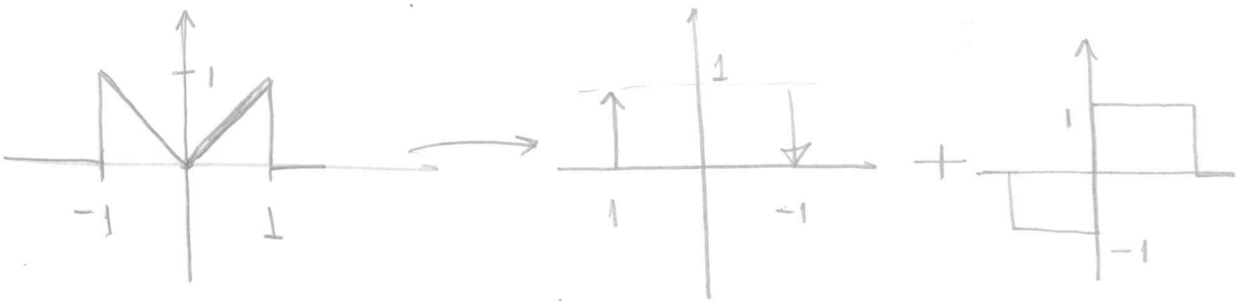
$\omega_{máx} = 10$

$\boxed{T < \frac{\pi}{12}}$

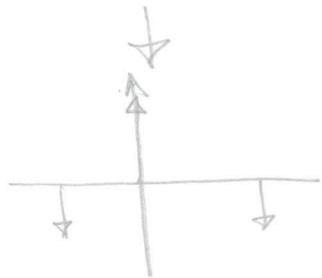
$B = \frac{\omega_{max}}{2\pi} \rightsquigarrow T < \frac{1}{2B}$

b) $p(t) = |t|\, G_2(t)$ $\quad T = 10 \quad \omega_0 = \frac{2\pi}{10} = \pi/5$

$$H(j\omega) = \frac{T\, G_{\omega_0}(\omega)}{P(\omega)}$$



$$P(\omega) = \frac{1}{j\omega}\left(e^{j\omega} - e^{-j\omega} + \frac{2 - e^{j\omega} + e^{-j\omega}}{j\omega} \right)$$



$$\boxed{H(j\omega) = \frac{10\, G_{\pi/5}(\omega)}{1/j\omega \left(e^{j\omega} - e^{-j\omega} + \frac{2 - e^{j\omega} - e^{-j\omega}}{j\omega} \right)}}$$ //

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$x(t) = 5t^3 \exp(3t)\, u(-t) - 4t \exp(-2t)\, u(t)$

$\mathcal{L}\left\{ \frac{t^m}{m!} e^{-at} u(t) \right\} = \frac{1}{(s+a)^{m+1}} \quad Re(s+a) > 0$

$\mathcal{L}\left\{ \frac{(-t)^m}{m!} e^{at} u(-t) \right\} = \frac{(-1)^{m+1}}{(s-a)^{m+1}} \quad Re(s-a) < 0 \quad \boxed{-2 < Re(s) < 3}$ //

$$X(\omega) = \frac{-4}{(s+2)^2} - \frac{5.6}{(s-3)^4} \Rightarrow \boxed{X(\omega) = \frac{-4}{(s+2)^2} - \frac{30}{(s-3)^4}}$$ //

(7) $X(s) = \dfrac{-3s^2 - s - 13}{(s+1)^2(s-2)}$ $\quad -1 < Re(s) < 2$

$$X(s) = \underbrace{\frac{A}{(s+1)^2}} + \frac{B}{(s+1)} + \frac{C^{=-3}}{(s-2)}$$

$$A(s-2) + B(s+1)(s-2) - 3(s+1)^2$$

$$B - 3 = -3 \longrightarrow \boxed{B = 0}$$

$$A - B - 6 = -1$$

$$\boxed{A = 5}$$

$$\mathcal{F}\left\{\frac{t^m}{m!} e^{-at} u(t)\right\} = \frac{1}{(s+a)^{m+1}}$$

$$X(s) = \underbrace{\frac{5}{(s+1)^2}}_{>-1} - \underbrace{\frac{3}{(s-2)}}_{<2}$$

$$\boxed{x(t) = 5te^{-t} u(t) + 3e^{2t} u(-t)}$$

(8) $I = \int t h(t) dt \quad h(t) = (t \exp(-2t)\, u(t)) * (\exp(-3t)\, u(t))$

$$H(s) = \frac{1}{(s+2)^2} \cdot \frac{1}{(s+3)}$$

$$I = -1 \frac{d}{ds} H(s) \Big|_{s=0}$$

$$J = +1 \cdot \left.\frac{2(s+2)(s+3)+(s+2)^2}{(s+2)^4(s+3)^2}\right|_{s=0}$$

$$J = \frac{\cancel{2}\cdot\cancel{2}\cdot 3+\cancel{4}}{2^4\cdot 9} = \frac{4}{4\cdot 9} = \frac{1}{9}$$

$$\boxed{J = 1/9}$$ //

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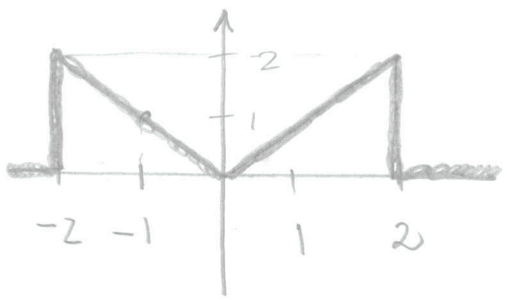
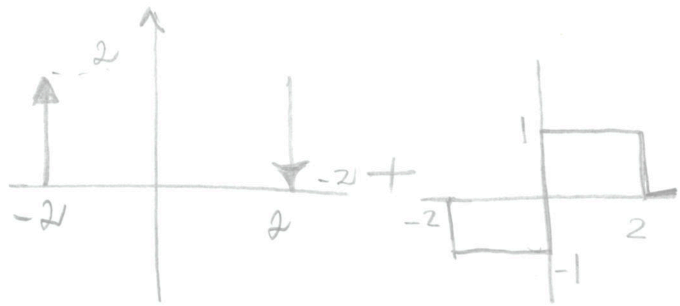
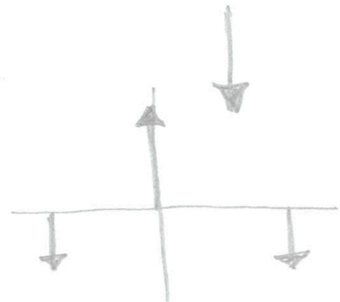
$x(t) = -t(G_2(t+1)) + tG_2(t-1)$



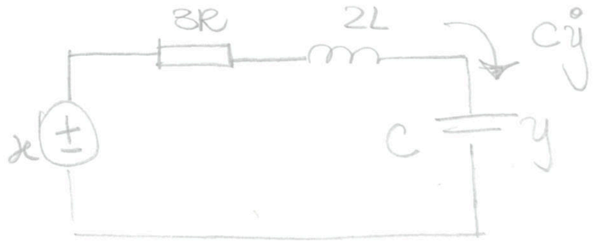




$$\boxed{X(s) = \frac{1}{s}\left(2e^{2s} - 2e^{-2s} + \frac{-e^{2s} + 2 - e^{-2s}}{s}\right)}$$ //

$$\boxed{s = \mathbb{C}}$$

(10)



$$x = 3RC\dot{y} + 2LC\ddot{y} + y$$

$$\frac{y}{x} = \frac{1/2LC}{\ddot{s} + \frac{3R}{2L}\dot{s} + \frac{1}{2LC}} = \frac{\omega_c^2}{\ddot{s} + \sqrt{2}\omega_c\dot{s} + \omega_c^2}$$

$$\frac{3R}{2L} = \sqrt{2}\omega_c \longrightarrow L = \frac{3R}{2\sqrt{2}\omega_c} \rightarrow \boxed{L = \frac{3R\sqrt{2}}{4\omega_c}}$$

$$\frac{1}{2LC} = \omega_c^2 \longrightarrow C = \frac{1}{2\omega_c^2}\,\frac{4\omega_c}{3R\sqrt{2}} =$$

$$\boxed{C = \frac{\sqrt{2}}{3R\omega_c}}$$