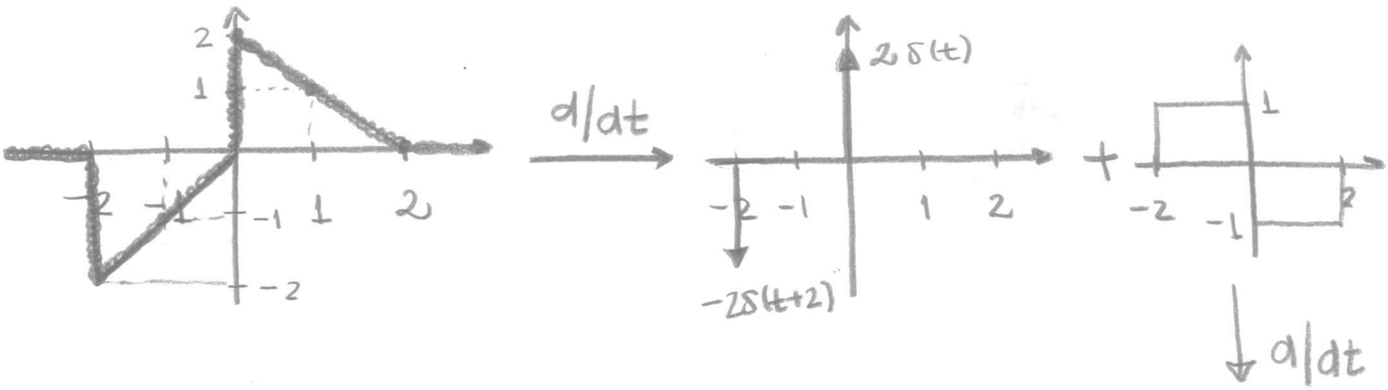
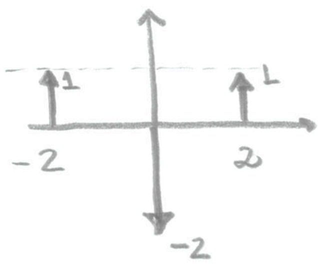
Resolução PR3 - 2016

① $x(t) = t\,G_2(t+1) + (2-t)\,G_2(t-1) \rightarrow X(\omega) = ?$



$$\boxed{X(\omega) = \frac{1}{j\omega}\left(-2e^{2j\omega} + 2 + \frac{e^{2j\omega} - 2 + e^{-2j\omega}}{j\omega}\right)}$$



② $\mathcal{F}\left\{\frac{18t}{(t^2+4)^2}\right\}$

Propriedades

$$\mathcal{F}\{e^{-a|t|}\} = \frac{2a}{\omega^2+a^2} \quad (1)$$

1° → $\mathcal{F}\{e^{-2|t|}\} = \frac{4}{\omega^2+2^2}$

$$\mathcal{F}\{X(t)\} = 2\pi x(-\omega) \quad (2)$$

2° → $\mathcal{F}\{t\,e^{-2|t|}\} = j\frac{d}{d\omega}\left(\frac{4}{\omega^2+4}\right) \leftarrow \mathcal{F}\{t^m x(t)\} = j^m \frac{d^m}{d\omega^m} X(\omega) \quad (3)$

$$\mathcal{F}\{t\,e^{-2|t|}\} = j\,\frac{-4(2\omega)}{(\omega^2+4)^2} = \frac{-8j\omega}{(\omega^2+4)^2}$$

Aplicando a prop (2) em $\mathcal{F}\{\underbrace{t e^{-2|t|}}_{x(t)}\} = \underbrace{\frac{-8j\omega}{(\omega^2+4)^2}}_{X(\omega)}$

$\mathcal{F}\{X(t)\} = 2\pi\, x(-\omega)$

$$\Rightarrow \mathcal{F}\left\{\frac{-8jt}{(t^2+4)^2}\right\} = 2\pi\left(-\omega\, e^{-2|\omega|}\right)$$

$X(t) = \dfrac{-8jt}{(t^2+4)^2}$

$x(-\omega) = -\omega e^{-2|\omega|}$

$$\Downarrow$$

$$\mathcal{F}\left\{\frac{-8jt}{(t^2+4)^2}\right\} = -2\pi\omega e^{-2|\omega|}$$

$$\Downarrow$$

$$\boxed{+\mathcal{F}\left\{\frac{+8jt}{(t^2+4)^2}\right\} = +2\pi\omega e^{-2|\omega|}}$$

3 $I = \displaystyle\int_{-\infty}^{\infty} Sa^2(3t)\, Sa(5t)\, dt$

Propriedade:

$\displaystyle\int_{-\infty}^{\infty} x(t)\, dt = X(0)$

$I = \mathcal{F}\{Sa^2(3t)\cdot Sa(5t)\}\Big|_{\omega=0}$

$\mathcal{F}\{x(t)y(t)\} = \dfrac{1}{2\pi} X(\omega) * Y(\omega)$

$I = \dfrac{1}{2\pi}\mathcal{F}\{Sa^2(3t)\} * \mathcal{F}\{Sa(5t)\}\Big|_{\omega=0}$

$\mathcal{F}\{TRI_{2T}(t)\} = T\,Sa^2\left(\frac{\omega T}{2}\right)$

Aplicando $\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$

$I = \dfrac{1}{2\pi}\left(\dfrac{2\pi}{6} TRI_{12}(\omega)\right) * \left(\dfrac{2\pi}{10} G_{10}(\omega)\right)\Big|_{\omega=0}$

$\mathcal{F}\{Sa^2\left(\frac{\omega T}{2}\right)\} = \dfrac{2\pi}{T} TRI_{2T}(\omega)$

$\mathcal{F}\{Sa\left(\frac{\omega_0 T}{2}\right)\} = \dfrac{2\pi}{\omega_0} G_{\omega_0}(\omega)$

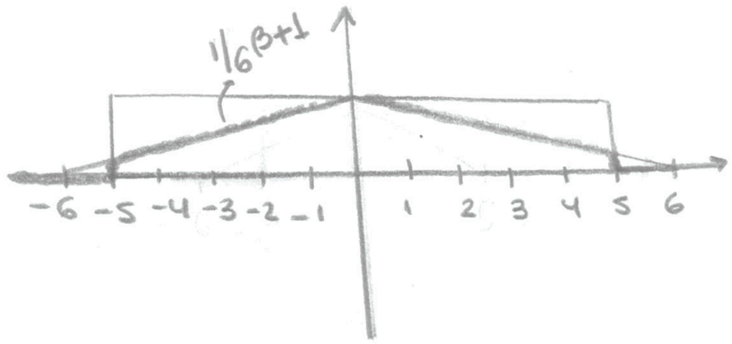
$\downarrow$

pela definição de convolução

$\downarrow$

$$I = \frac{1}{2\pi}\cdot\frac{2\pi}{6}\cdot\frac{2\pi}{10}\left(\int_{-\infty}^{\infty} TRI_{12}(\beta)\, G_{10}(\omega-\beta)\, d\beta\right)\Bigg|_{\omega=0}$$

$$I = \frac{\pi}{30}\underbrace{\int_{-\infty}^{\infty} \mathrm{TRI}_{12}(\beta)\, G_{10}(-\beta)\, d\beta}_{} = 2\frac{\pi}{30}\int_{-5}^{0}\left(\tfrac{1}{6}\beta + 1\right) d\beta$$



$$\frac{\pi}{15}\left(\frac{\beta^2}{12} + \beta\right)\Big|_{-5}^{0}$$

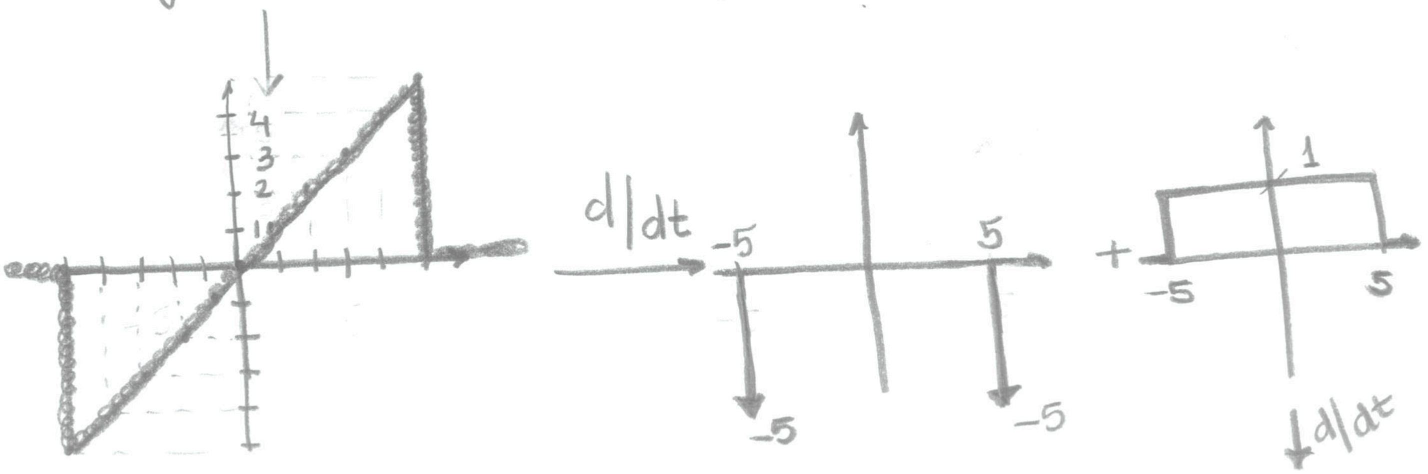
$$\frac{\pi}{15}\left(-\left(\frac{25}{12} - 5\right)\right)$$

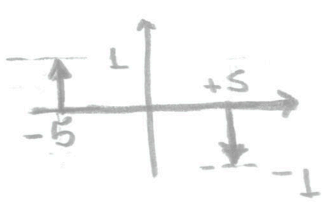
$$\frac{\pi}{15}\left(-\frac{25}{12} + \frac{60}{12}\right) = \frac{\cancel{35}^{\,7}\pi}{\cancel{15}_{\,3}\cdot 12}$$

$$\boxed{I = \frac{7\pi}{36}}$$

(4) $X(\omega) = j\omega\, G_{10}(\omega) \longrightarrow x(t) = ?$

$\mathcal{F}\{jt\, G_{10}(t)\} = 2\pi x(-\omega) =$





$$X(t\,G_{10}(t)) = j\,\frac{1}{j\omega}\left(-5e^{5j\omega} - 5e^{-5j\omega} + \frac{e^{5j\omega} - e^{-5j\omega}}{j\omega}\right)$$

$$2\pi x(-\omega) = \frac{1}{\omega}\left(-5e^{5j\omega} + 5e^{-5j\omega} + \frac{e^{5j\omega} - e^{-5j\omega}}{j\omega}\right)$$

$$x(-\omega) = \frac{1}{2\pi(\omega)}\left(-5e^{5j\omega} + 5e^{-5j\omega} + \frac{e^{5j\omega} - e^{-5j\omega}}{j\omega}\right)$$

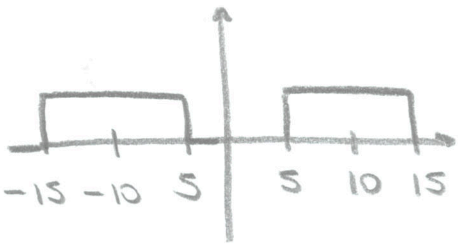
$$\downarrow$$

$$x(t) = \frac{1}{2\pi(-t)}\left(-5e^{-5jt} + 5e^{+5jt} + \frac{e^{-5jt} - e^{5jt}}{-jt}\right)$$

$$\boxed{x(t) = \frac{1}{2\pi t}\left(5e^{-5jt} + 5e^{-5jt} + \frac{e^{-5jt} - e^{+5jt}}{jt}\right)}$$

5

a) $X(\omega) = G_{10}(\omega+10) + G_{10}(\omega-10)$



$\omega_M = 15$

$2\pi B = \omega_M$

$B = \dfrac{15}{2\pi}$

$$T < \frac{1}{2B}$$

$$\downarrow$$

$$T < \frac{1}{\frac{15}{\pi}} \rightarrow \boxed{T < \frac{\pi}{15}}$$

b) Máxima freq $\Rightarrow \omega_{máx} = \dfrac{\pi}{10}$ rad/s

$x_a(t) = \sum x(k8)\, p(t-k8)$

$p(t) = TRI_3(t)$

$T = 8$, $\omega_0 = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$

$$H(j\omega) = \frac{T\, G_{\omega_0}(\omega)}{P(\omega)}$$

$P(\omega) = \mathcal{F}\{p(t) = TRI_3(t)\} \longrightarrow \mathcal{F}\{TRI_{2T}\} = T\, Sa^2\left(\frac{\omega T}{2}\right)$

$P(\omega) = \frac{3}{2}\, Sa^2\left(\frac{\omega 3}{4}\right)$

$\longrightarrow \boxed{H(j\omega) = \frac{8 \cdot G_{\pi/4}(\omega)}{3/2\, Sa^2(3\omega/4)}}$

⑥ $x(t) = 3t^3 e^{-5t} \mu(-t)$

Propriedades

$\mathcal{L}\{x(-t)\} = X(-s) \quad (1)$

$\mathcal{L}\{t^m/m!\, e^{-at} \mu(t)\} = \frac{1}{(s+a)^{m+1}} \quad (2)$

Usando (1), temos

$y(t) = x(-t) = -3t^3 e^{5t} \mu(t)$

$Y(s) \Rightarrow$ aplicando (2)

$Y(s) = \mathcal{L}\{-3t^3 e^{5t} \mu(t)\} = -3! \cdot 3 \frac{1}{(s-5)^4} = \frac{-18}{(s-5)^4}$

$Y(s) = X(-s) \longrightarrow \boxed{X(s) = \frac{-18}{(s+5)^4}}$

$Re(s+5) < 0$

$\boxed{Re(s) < -5}$

7

$$X(s) = \frac{5s^2+13s-24}{(s+2)^2(s-4)} \quad -2 < Re(s) < 4$$

$$X(s) = \frac{A}{(s+2)^2} + \frac{B}{(s+2)} + \frac{C}{(s-4)}$$

$$A(s-4) + B\underbrace{(s+2)(s-4)}_{s^2-2s-8} + C\underbrace{(s+2)^2}_{s^2+4s+4}$$

$$\begin{cases} B+C=5 \\ A-2B+4C=13 \\ -4A-8B+4C=-24 \end{cases} \qquad C = \frac{5(4)^2+13.4-24}{(4+2)^2} = 3$$

$$\boxed{B=2} \qquad A = 13+4-12$$

$$\boxed{A=5}$$

$$X(s) = \underbrace{\frac{5}{(s+2)^2} + \frac{2}{(s+2)}}_{Re(s)>-2} + \underbrace{\frac{3}{(s-4)}}_{Re(s)<4}$$

$x(t) \Rightarrow$ Propriedades

$$\mathcal{L}\{t^m/m!\, e^{-at}\mu(t)\} = \frac{1}{(s+a)^{m+1}}$$

$$\mathcal{L}\{(-t)^m/m!\, e^{at}\mu(-t)\} = \frac{1}{(-s+a)^{m+1}} = \frac{(-1)^{m+1}}{(s-a)^{m+1}}$$

$$\boxed{x(t) = (5.t\, e^{-2t} + 2e^{-2t})\mu(t) - 3e^{4t}\mu(-t)}$$

8 $I = \int_{-\infty}^{\infty} t h(t) \, dt$ sendo h(t) a resposta ao impulso

$$\dot{v}_1 = v_2 \longrightarrow v_2 p = p^2 v_1$$
$$\dot{v}_2 = -13v_1 - 4v_2 + x$$
$$y = 2v_1 + v_2$$

Notação $\dot{v}_1 = p v_1$

$$p^2 v_1 + 4p v_1 + 13 v_1 = x$$

$$v_1 = \frac{x}{p^2+4p+13} \longrightarrow v_2 = p\, v_1 = \frac{p x}{p^2+4p+13}$$

$$y = \frac{2x}{p^2+4p+13} + \frac{px}{p^2+4p+13}$$

$$\frac{y}{x} = \frac{(p+2)}{p^2+4p+13} \Rightarrow \boxed{H(s) = \frac{s+2}{s^2+4s+13}}$$

$$\boxed{I = -\frac{5}{132}}$$

Propriedade

$$\int_{-\infty}^{\infty} x(t)\, dt = X(s)\Big|_{s=0}$$

$$\mathcal{L}\{t x(t)\} = -\frac{d X(s)}{ds}$$

$$I = -\frac{[(13) - 8]}{132}$$

$$\int_{-\infty}^{\infty} t h(t)\, dt = -\frac{d}{ds} H(s)\Big|_{s=0} = -\frac{(s^2+4s+13) - (s+2)(2s+4)}{(s^2+4s+13)^2}\Bigg|_{s=0}$$

9 $x(t) = (1 - e^{2t})\, G_4(t-2)$

$x(t) = (1 - e^{2t})\big(u(t) - u(t-4)\big)$

$x(t) = (1 - e^{2t})\, u(t) - u(t-4)(1 - e^{2t})$

Propriedades

$\mathcal{L}\{u(t)\} = 1/s \quad (1)$

$\mathcal{L}\{x(t-\tau)\} = X(s)e^{-s\tau}$

$\mathcal{L}\{e^{\lambda t} u(t)\} = \dfrac{1}{s-\lambda} \quad (2)$

$\mathcal{L}\{e^{-at} x(t)\} = X(s+a) \quad (3)$

$\mathcal{L}\{u(t-\tau)\} = \dfrac{e^{-s\tau}}{s} \quad (5)$

$\mathcal{L}\{e^{-at} u(t-\tau)\} = \dfrac{e^{-\tau(s+a)}}{(s+a)} \quad (4)$

Aplicando as propriedades acima na expressão

$$x(t) = \underset{\downarrow\atop 1}{u(t)} - \underbrace{e^{2t}u(t)}_{\downarrow\atop 2} - \underbrace{u(t-4)}_{5} + \underbrace{1e^{2t}u(t-4)}_{4}$$

$$\frac{1}{s} - \frac{1}{s-2} - \frac{e^{-4s}}{s} + \frac{e^{-4(s-2)}}{s-2} \qquad Re(s) = \mathbb{C}$$

$$\boxed{x(t) = \frac{1}{s}\left(1 - e^{-4s}\right) + \frac{1}{s-2}\left(e^{-4(s-2)} - 1\right)}$$

(10) $(p^3 L_1 C_2 L_3 + p^2 2 L_1 C_2 R + p(L_1+L_3) + 2R)y = 2Rx$

$$\frac{y}{x} = \frac{2R}{p^3 L_1 C_2 L_3 + p^2 2 L_1 C_2 R + p(L_1+L_3) + 2R}$$

Dividindo tudo por $2R$

$p = \lambda$

$$\frac{y}{x} = \frac{1}{p^3 \frac{L_1 C_2 L_3}{2R} + p^2 L_1 C_2 + p\left(\frac{L_1+L_3}{2R}\right) + 1}$$

Trocando $p \to \lambda$

$$H(s) = \frac{1}{\lambda^3\left(\frac{L_1 C_2 L_3}{2R}\right) + \lambda^2 (L_1 C_2) + \lambda\left(\frac{L_1+L_3}{2R}\right) + 1}$$

$$D(\lambda) = \lambda^3 + 2\lambda^2 + 2\lambda + 1 \qquad \lambda = \frac{s}{\omega_c}$$

$$\frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3} = H(s) = \frac{2R/L_1 C_2 L_3}{\lambda^3 + \lambda^2\left(\frac{2R}{L_3}\right) + \lambda\left(\frac{L_1+L_3}{L_1 C_2 L_3}\right) + \frac{2R}{L_1 C_2 L_3}}$$

$$\begin{cases} \omega_c^3 = \dfrac{2R}{L_1 C_2 L_3} \rightarrow L_1 C_2 L_3 = \dfrac{2R}{\omega_c^3} \\ \dfrac{2R}{L_3} = 2\omega_c \rightarrow \boxed{L_3 = \dfrac{R}{\omega_c}} \\ \dfrac{L_1+L_3}{L_1 C_2 L_3} = 2\omega_c^2 \end{cases}$$

$$\frac{L_1 + R/\omega_c}{2R/\omega_c^3} = 2\omega_c^2$$

$$L_1 + R/\omega_c = \frac{4R}{\omega_c}$$

$$\boxed{L_1 = \frac{3R}{\omega_c}}$$

$$L_1 C_2 L_3 = \frac{2R}{\omega_c^3}$$

$$\downarrow$$

$$C_2 = \frac{2\cancel{R}}{\omega_c \cancel{^3}} \cdot \frac{\cancel{\omega_c}}{3\cancel{R}} \cdot \frac{\cancel{\omega_c}}{R}$$

$$\boxed{C_2 = \frac{2}{3\omega_c R}}$$