RESOLUÇÃO PR2 - 2S - 2014

1 $x(t) = (t+1)\,G_1(t+1/2) + G_1(t-0{,}5)$, determinar e esboçar $x(2-t/2)$

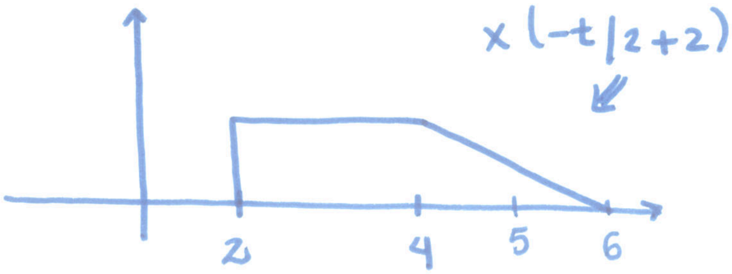
$\downarrow$

$2-t/2$

$$x(t/2+2) = (\underbrace{(-t/2+2)+1}_{-t/2+3})\,G_2(\underbrace{(2-t/2)+1/2}_{2{,}5-t/2\,=\,\frac{5-t}{2}}) + G_2(\underbrace{(2-t/2)-1/2}_{1{,}5-t/2\,=\,\frac{3-t}{2}})$$

$$\boxed{x(-t/2+2) = (3-t/2)\,G_2(t-5) + G_2(t-3)}$$

Como $t/2 \rightarrow$ haverá uma expansão $\rightarrow G_{1\times 2} = G_2$



2 $y(t) = (\sin^2(2t))\,x(t)$ linear = ? invariante no tempo = ?

$$g\{a_1x_1(t) + b_2x_2(t)\} = a_1\,g\{x_1(t)\} + b_2\,g\{x_2(t)\}$$

$$g\{a_1 . x_1(t) + b_2x_2(t)\} = a_1g\{x_1(t)\} + b_2\,g\{x_2(t)\} \quad //$$

linear

$$y(t-a) = x(t-a)$$

$$y(t-a) = (\sin^2(2(t-a))\,x(t-a)) \neq x(t-a)$$

variante no tempo

③ $x(t) = 2e^{-10t} \rightsquigarrow y_f(t) = ?$ $\quad y(t) = x(t-10)$

Para determinar $y_f(t)$ devemos determinar 1º $\to H(s)$

Pela teoria assumindo $x(t) = e^{st}$ e $y(t) = H(s)e^{st}$

$$y(t) = x(t-10) \Rightarrow H(s)e^{st} = e^{s(t-10)}$$

$$H(s) = \frac{e^{st} \cdot e^{-10s}}{e^{st}}$$

$$H(s) = e^{-10s}$$

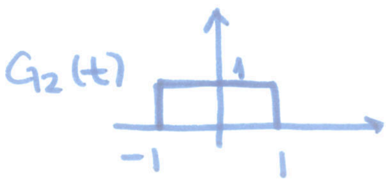
$y_f(t) \to$ para $x(t) = 2e^{-10t}$

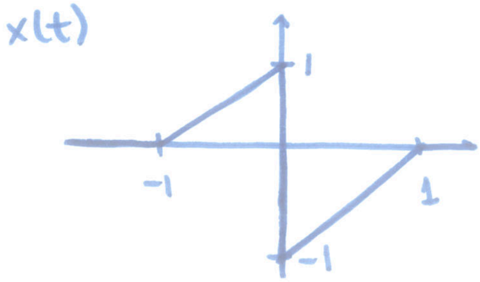
s

$$y_f(t) = \underbrace{2e^{-10t}}_{x(t)} \cdot H(-10) = 2e^{-10t} \cdot e^{100}$$

$$\boxed{y_f(t) = 2 \cdot e^{-10(t-10)}}$$ //

④ Determine e esboce a convolução de $x(t) = (t+1)G_1(t+0,5) + (t-1)G_1(t-0,5)$ com $G_2(t)$



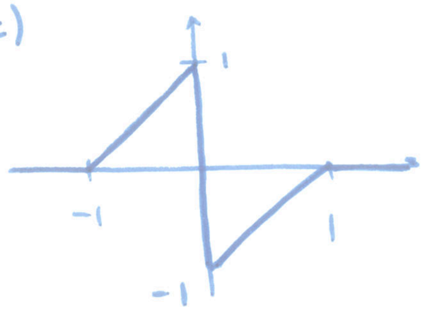


$x(t) * u(t) = I_x(t)$

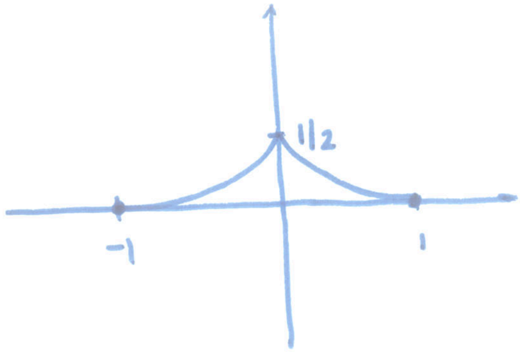
$G_2(t) = u(t+1) - u(t-1)$

$$x(t) * G_2(t) = \underbrace{x(t) * u(t+1)}_{I_x(t+1)} - \underbrace{u(t-1) * x(t)}_{I_x(t-1)}$$

x(t)



$I_x(t)$ $\longrightarrow$



$$\int_{-1}^{t} (\tau+1)\,d\tau = \frac{\tau^2}{2}+\tau \Big|_{-1}^{t} \qquad \int_{0}^{t} (-1+\tau)\,d\tau = \frac{\tau^2}{2}-\tau \Big|_{0}^{t}$$

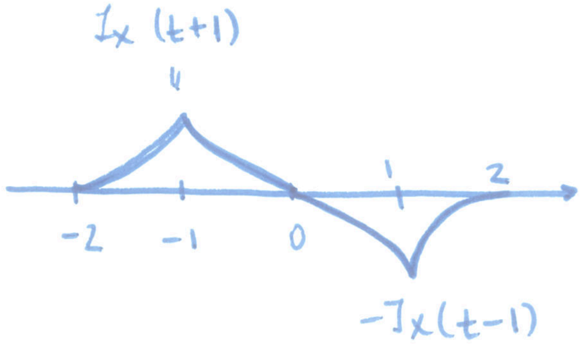
$$\frac{t^2}{2}+t-\frac{1}{2}+1 \qquad \frac{t^2}{2}-t+\frac{1}{2} = I_x(t)\,G_1(t-0{,}5)$$

$$\int_{-1}^{0} (\tau+1)\,d\tau$$

$$\frac{t^2}{2}+t+1/2 = I_x(t)\,G_1(t+0{,}5)$$

$$\rightarrow I_x(t) = \left(\frac{t^2}{2}+t+\frac{1}{2}\right) G_1(t+0{,}5) + \left(\frac{t^2}{2}-t+\frac{1}{2}\right) G_1(t-$$

$$x(t) * G_2(t) =$$



$$x(t) * G_2(t) = I_x(t+1).\,G_1(t+1{,}5) - I_x(t-1).\,G_1(t-1{,}5)$$

$$\left(\frac{(t+1)^2}{2}+t+1+\frac{1}{2}\right) G_1(t+1{,}5) - \left(\frac{(t-1)^2}{2}-(t-1)+\frac{1}{2}\right) G_1(t-1{,}5)$$

$$\boxed{\left(\frac{t^2}{2}+2t+2\right) G_1(t+1{,}5) - \left(\frac{t^2}{2}-2t+2\right) G_1(t-1{,}5) + + \frac{t^2}{2} G_1(t+0{,}5) - \frac{t^2}{2} G_1(t-0{,}5)}$$

5 a) $h(t) = ?$

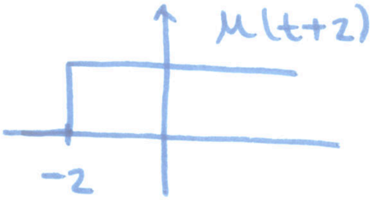
$$y(t) = \mathcal{G}\{x(t)\} = \int_{-\infty}^{\infty} \exp(\beta)\,\mu(\beta+2)\,x(t-\beta)\,d\beta$$

$$y(t) = h(t) = \mathcal{G}\{\delta(t)\} = \int_{-\infty}^{\infty} e^{\beta}\mu(\beta+2)\,\delta(t-\beta)\,d\beta$$

$$\boxed{h(t) = e^{t}\mu(t+2)}$$

b) causal? BIBO?

causal $\longrightarrow$ $h(t) = 0 \longrightarrow t < 0$



$\mu(-1) = 1$

$\therefore \mu(-1) \neq 0$

Não causal //

BIBO $\longrightarrow$ $\int |h(t)|\,dt < \infty$ $\quad \int_{-\infty}^{\infty} e^{t}\mu(t+2) = e^{t}\Big|_{-\infty}^{\infty}$

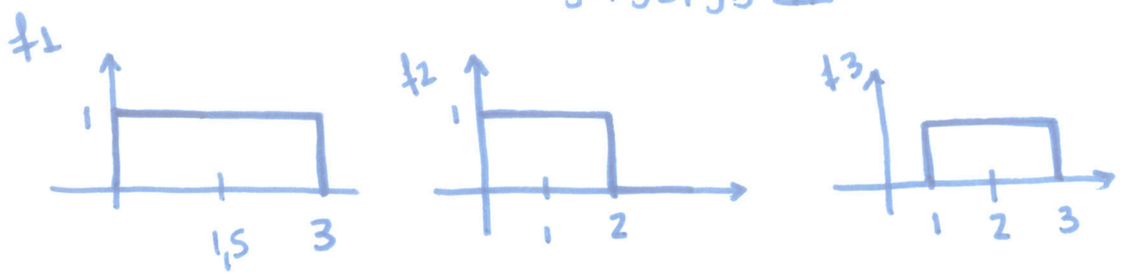
$$e^{\infty} - \underbrace{e^{-\infty}}_{0} = e^{\infty} > +\infty$$

$\therefore$

<u>NÃO BIBO</u>

⑥ $f_1(t) = G_3(t-1{,}5)$ $f_2 = G_2(t-1)$ $f_3 = G_2(t-2)$

determine g_1, g_2, g_3 ⊥



Pela Regra de Gram-Schmidt

$$g_1 = f_1$$

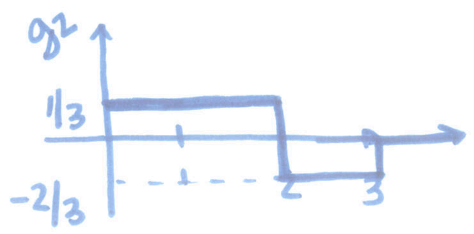
$$g_2 = f_2 - \frac{\langle g_1, f_2\rangle}{\langle g_1^2\rangle} g_1$$

$$g_3 = f_3 - \frac{\langle g_1, f_3\rangle}{\langle g_1^2\rangle} g_1 - \frac{\langle g_2, f_3\rangle}{\langle g_2^2\rangle} g_2$$

$$\langle g_1, f_2\rangle = \int_0^2 1 . dt = 2 \qquad \langle g_1, f_3\rangle = \int_0^1 0 . dt + \int_1^3 dt = 2$$

$$\langle g_1^2\rangle = \int_0^3 1 . dt = 3$$

$$\boxed{g_2 = f_2 - \frac{2}{3} g_1}$$

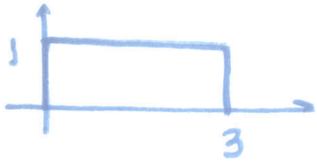


$$\langle g_2, f_3\rangle = \int_0^1 0 . 1/3\, dt + \int_1^2 1/3 . 1\, dt + \int_2^3 -2/3\, dt = \frac{2}{3} - \frac{1}{3} + \frac{4}{3} - \frac{6}{3} = -\frac{1}{3}$$

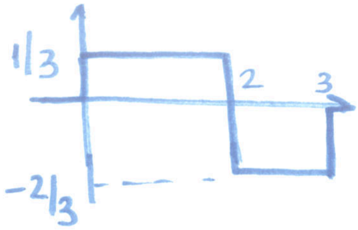
$$\langle g_2^2\rangle = \int_0^2 1/9\, dt + \int_2^3 +4/9 = \frac{2}{9} + \frac{12}{9} - \frac{8}{9} = +\frac{6}{9} = \frac{2}{3}$$

$$\boxed{g_3 = f_3 - \frac{2}{3} g_1 + \frac{1}{2} . g_2}$$

$g_1 = f_1 = G_3(t-1{,}5)$

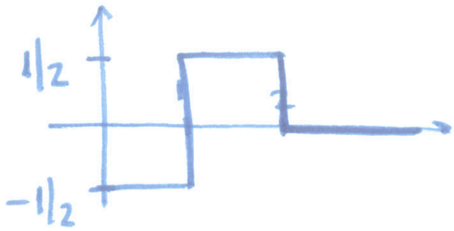


$g_2 = f_2 - \frac{2}{3} g_1 = G_2(t-1) - \frac{2}{3} G_3(t-1{,}5)$



$g_3 = f_3 - \frac{2}{3} g_1 + \frac{1}{2} g_2 = G_2(t-2) - \frac{2}{3} G_3(t-1{,}5) +$

$+ 1/2 \left(G_2(t-1) - 2/3\, G_3(t-1{,}5) \right)$

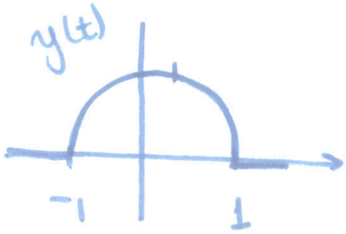


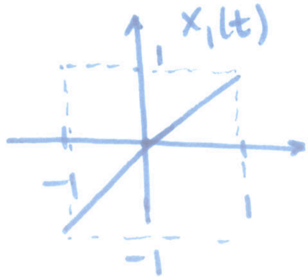
$-\frac{2}{3} + \frac{1}{6} = \frac{-3}{6} = -\frac{1}{2}$

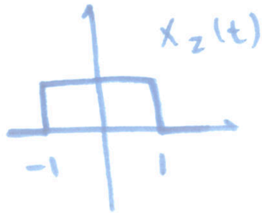
$\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

7

$$\epsilon(t) = \underbrace{t^2 G_2(t)}_{y(t)} - \left(a \underbrace{t\, G_2(t)}_{x_1(t)} + b \underbrace{G_2(t)}_{x_2(t)}\right)$$







$$\begin{bmatrix} \langle x_1^2 \rangle & \langle x_1 x_2 \rangle \\ \langle x_1 x_2 \rangle & \langle x_2^2 \rangle \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \langle y x_1 \rangle \\ \langle y x_2 \rangle \end{bmatrix}$$

$\langle x_1^2 \rangle = \int_{-1}^{1} t^2\, dt = \left.\frac{t^3}{3}\right|_{-1}^{1} = 2/3$

$\langle y x_1 \rangle = \int_{-1}^{1} t^3\, dt = \left.\frac{t^4}{4}\right|_{-1}^{1} = 0$

$\langle x_1 x_2 \rangle = \int_{-1}^{1} t\, dt = 0$

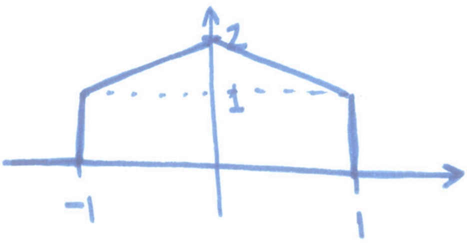
$\langle y x_2 \rangle = \int_{-1}^{1} t^2\, dt = \left.\frac{t^3}{3}\right|_{-1}^{1} = 2/3$

$\langle x_2^2 \rangle = \int_{-1}^{1} 1\, dt = 2$

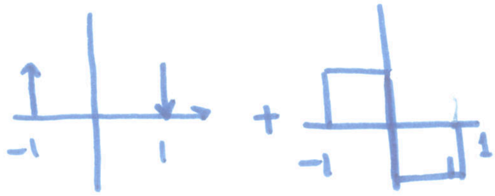
$$\begin{bmatrix} 2/3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2/3 \end{bmatrix} \Rightarrow \frac{2}{3}a = 0 \rightarrow \boxed{a = 0}$$

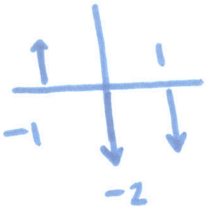
$$2b = \frac{2}{3} \rightarrow \boxed{b = 1/3}$$

8 a) c_k $\quad x(t) = \sum_{-\infty}^{+\infty} p(t-5k) \quad p(t) = (t+2)G_1(t+0{,}5) + (2-t)G_1(t-0{,}5)$



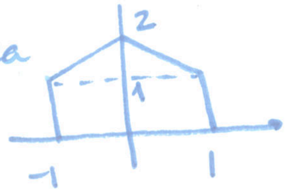
$\xrightarrow{d/dt}$





$$c_k = \frac{1}{5}\left[\frac{e^{j\frac{2\pi}{5}k} - e^{-j\frac{2\pi}{5}k} - 2}{\left(\frac{2\pi}{5}kj\right)} + \frac{e^{j\frac{2\pi}{5}k} - e^{-j\frac{2\pi}{5}k}}{\left(\frac{2\pi}{5}kj\right)^2}\right]$$

b) $c_0 = \text{valor médio} = \frac{\text{Área}}{T}$



$$\boxed{c_0 = \frac{3}{5}}$$

9 $x(t) = 2j\exp(j\pi t/3) + 3\cos(2\pi t/5)$

a) $T = ?$ $\quad x(t) = 2j\exp(j\pi t/3) + 3/2\left(\exp(2\pi t/5 j) + \exp(-2\pi t/5 j)\right)$

$$\frac{\pi}{3} = \frac{2\pi}{T} \rightarrow T_1 = 2.3 = 6$$

$$\frac{2\pi}{5} = \frac{2\pi}{T} \rightarrow T_2 = 5$$

$$6.\underset{5}{p} = 5.\underset{6}{q} = T$$

$$\boxed{T = 30}$$

$$\frac{2\pi}{30}\cdot 5 = \frac{\pi}{3} \qquad \frac{2\pi}{30}\cdot 6 = \frac{2\pi}{5}$$

$$x(t) = 2j e^{j\overset{=k}{5}\frac{2\pi}{30}t} + \frac{3}{2}\left(e^{j\overset{=k}{6}\cdot\frac{2\pi}{5}t} + e^{\overset{k}{-j6}\cdot\frac{2\pi}{5}t}\right)$$

$\boxed{C_5 = 2j}$ $\boxed{C_6 = C_{-6} = 3/2}$ Demas nulos

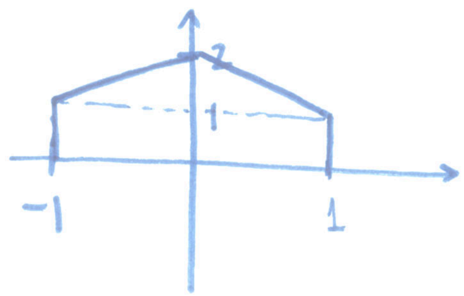
c) $P_m = \sum |c_k|^2 = 4 + \frac{9}{4} + \frac{9}{4} = 4 + \frac{18}{4} = 4 + \frac{9}{2} = \boxed{\frac{17}{2}}$

(10)

$P_m = ?$

$$x(t) = \sum_{-\infty}^{+\infty} p(t-5k) \qquad p(t) = (t+2)\,G_1(t+0{,}5) + (2-t)\,G_1(t-0{,}5)$$

$$P_m = \frac{1}{T}\int_T |x(t)|^2 dt$$



$$P_m = \frac{1}{5}\cdot 2\int_{-1}^{0} (t+2)^2\,dt = \frac{2}{5}\; \frac{t^3}{3} + 2t^2 + 4t\Big|_{-1}^{0}$$

$$\frac{2}{5}\left(\frac{1}{3} - 2 + 4\right) = \frac{2}{5}\left(\frac{1-6+12}{3}\right)$$

$$= \boxed{\frac{14}{15}}$$